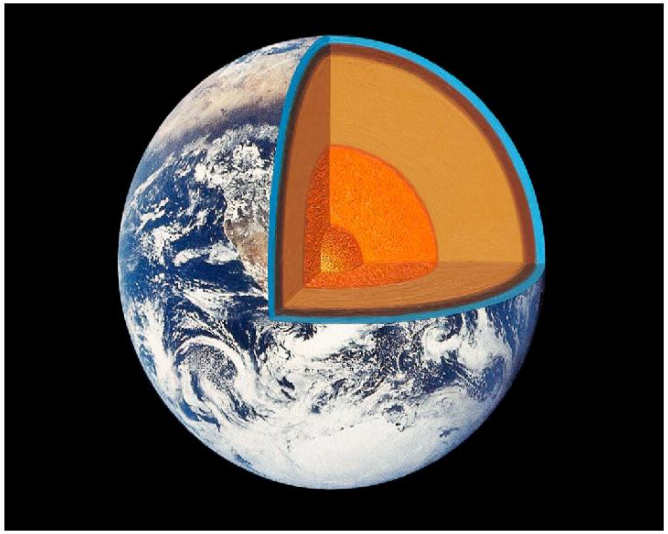
# Reversals of a large scale field generated over a turbulent background

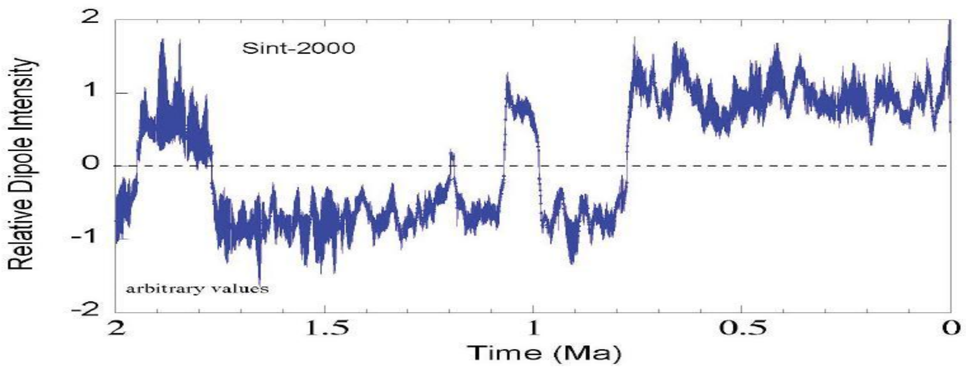
F. Pétrélis

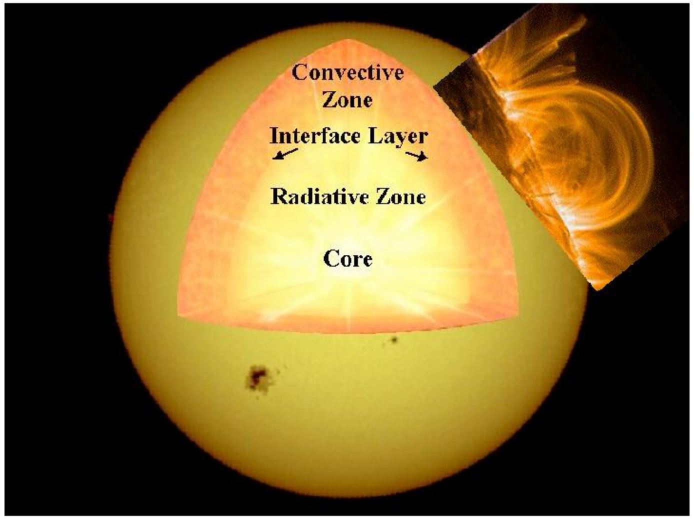
**Laboratoire de Physique Statistique, CNRS**

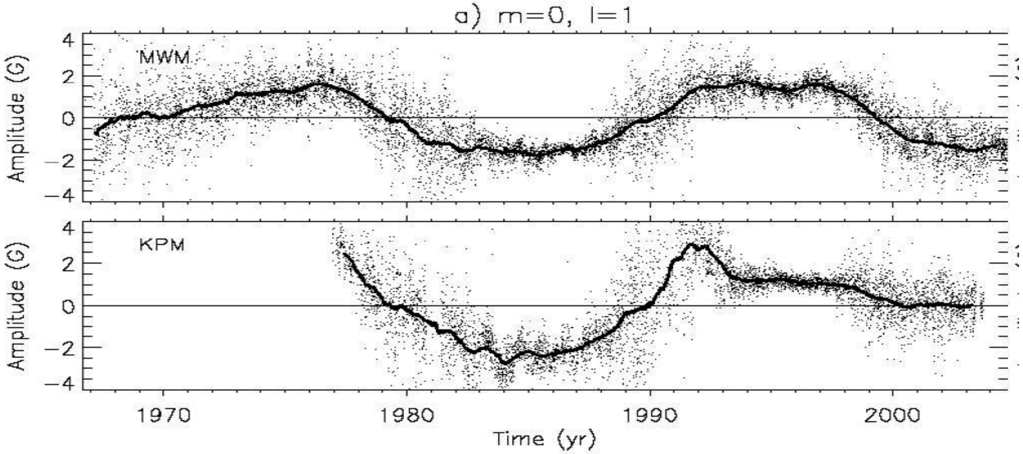
**Ecole Normale Supérieure, Paris, France**

# Reversing magnetic fields in astrophysical objects



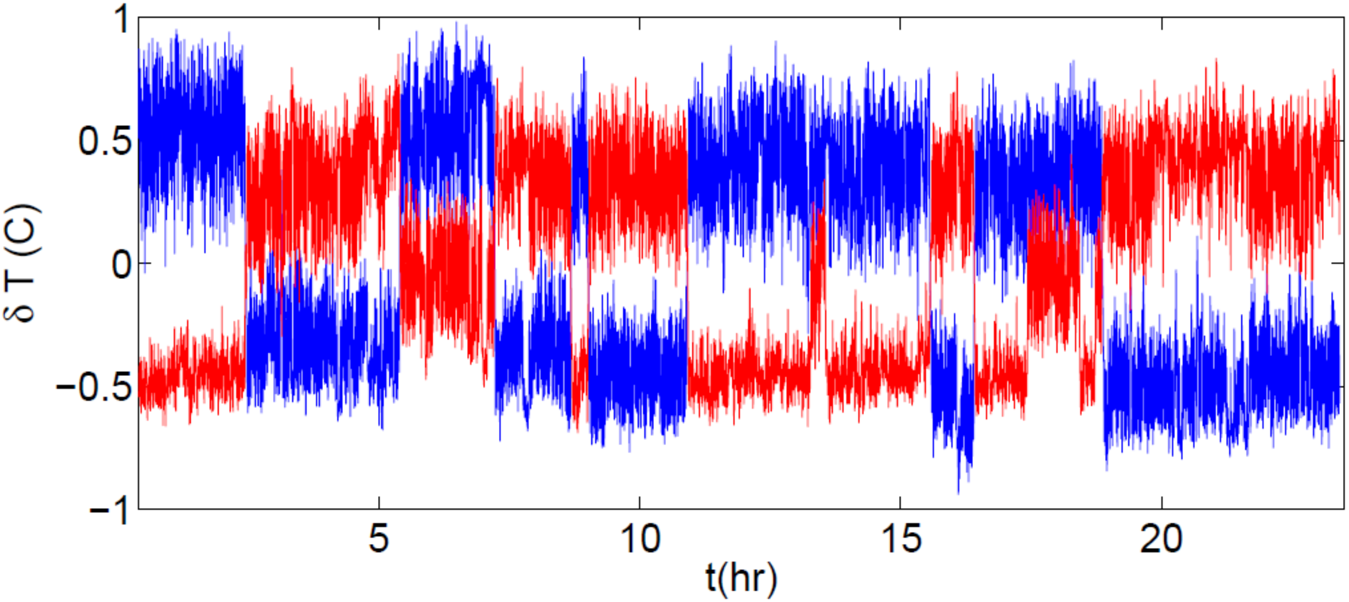



Highly turbulent flows, Re>>1

# Reversals of the large scale velocity in thermal convection (with C. Laroche, S. Fauve)

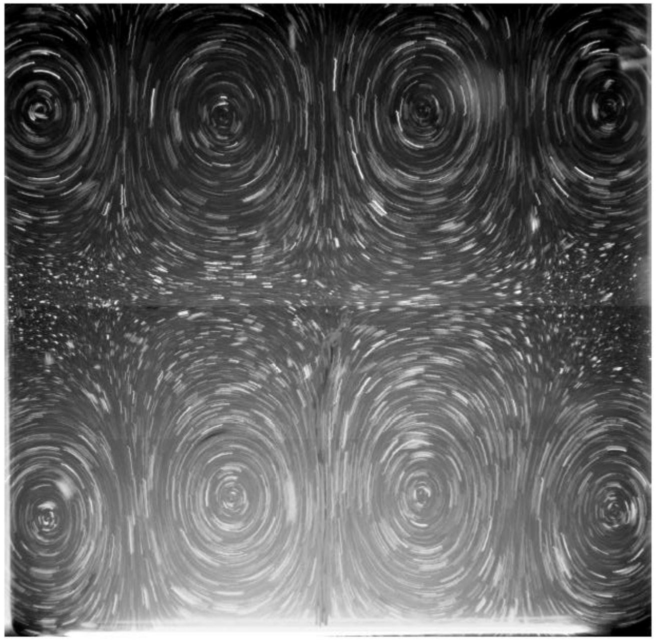


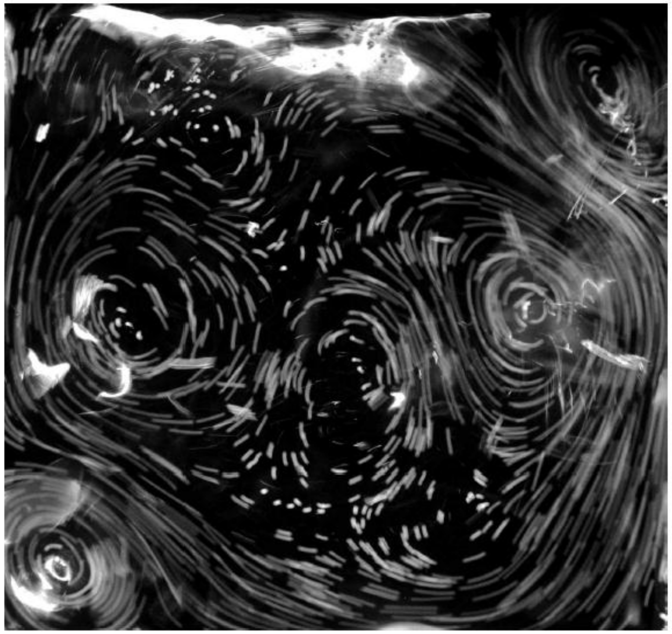

Highly supercritical Rayleigh Bénard convection of mercury in a square container

Many other observations (Liu and Zhang, Ahlers, Niemela, Sreenivasan, …)

# Large scale circulation in a 2D Kolmogorov flow
(J. Herault, G. Michel, B. Gallet, S. Fauve)

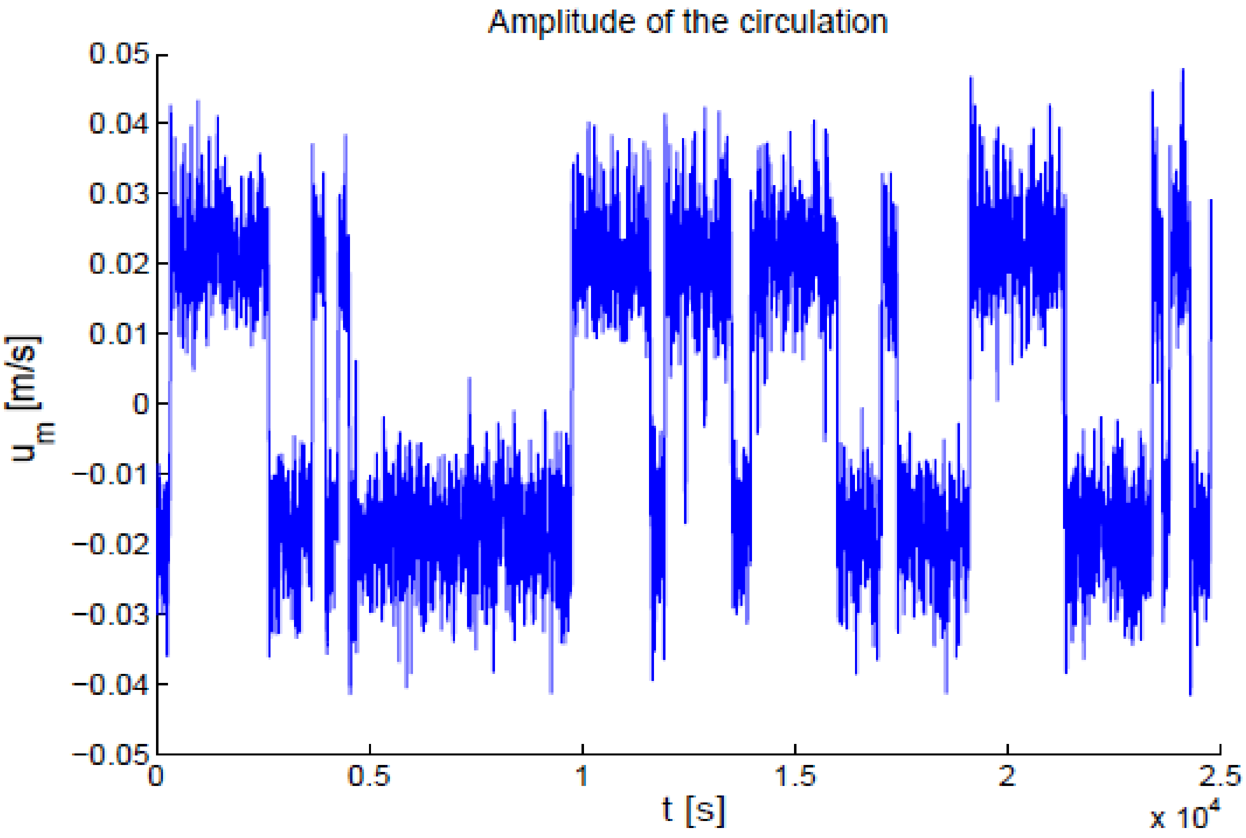
Exp (Sommeria 86): periodic electrical forcing (array of electrodes) in a liquid metal layer plunged into a vertical magnetic field





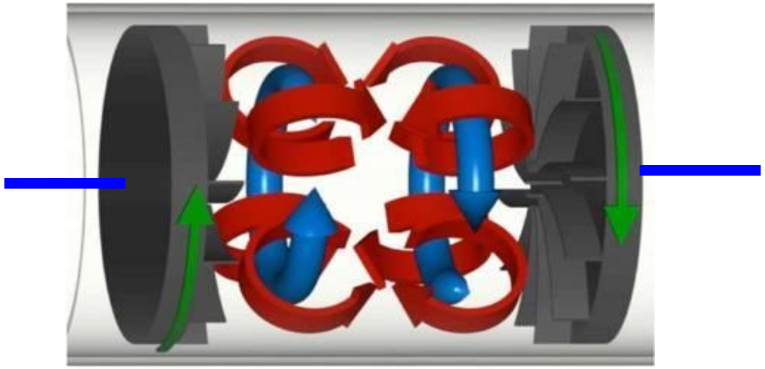
Forcing drives large scale circulation (2D inverse cascade)

The large scale circulation switches direction (random reversals)



# Some results from the Von Karman Sodium experiment:

with ENS-Lyon (**S. Mirales,** **G. Verhille, M. Bourgoin, P. Odier, J.-F. Pinton, N. Plihon)**
CEA-Saclay **(S. Aumaître, J. Boisson, A. Chiffaudel, B. Dubrulle, F. Daviaud)**
ENS (**B. Gallet, J. Herault,** **M. Berhanu, C. Gissinger, S. Fauve, N. Mordant, F. Pétrélis)**

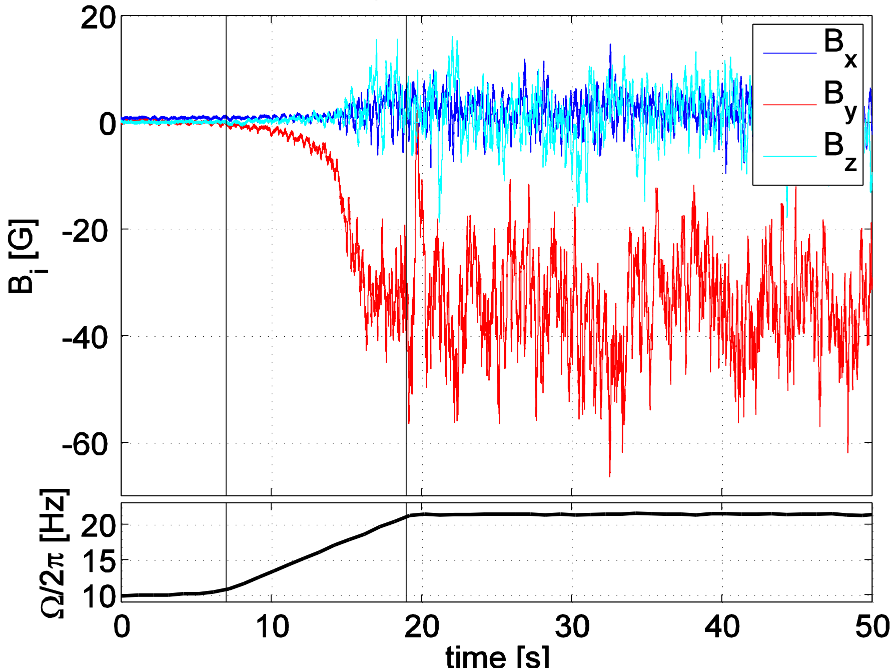


**150L liquid sodium**

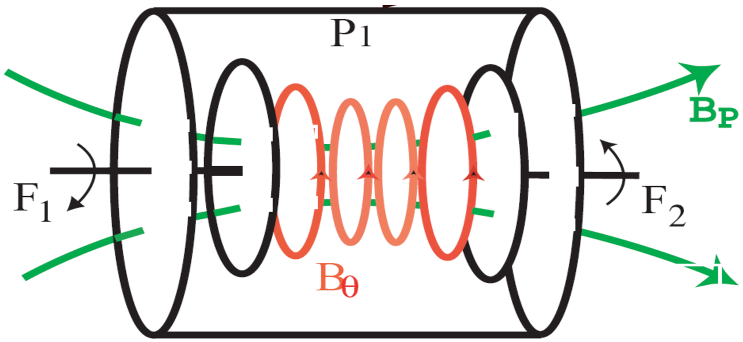
**P=300 KW, Re=10^6**

**Soft iron disks**

The Dynamo Effect:




In exact counter rotation:
Forcing is symmetric
Dominant field is an axial dipole



## Some results from the VKS experiment:

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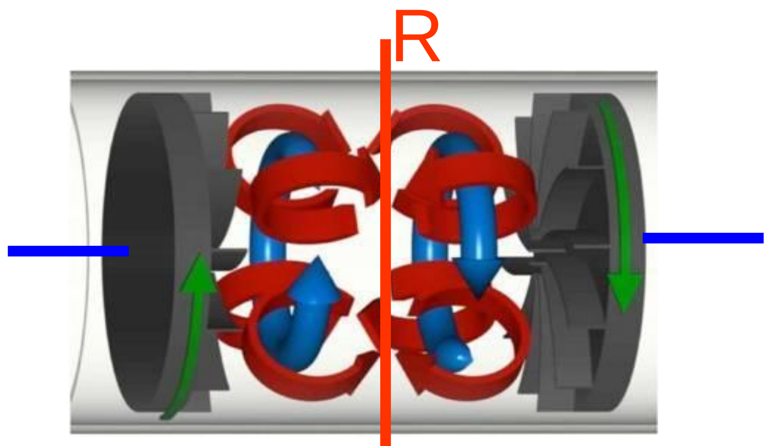
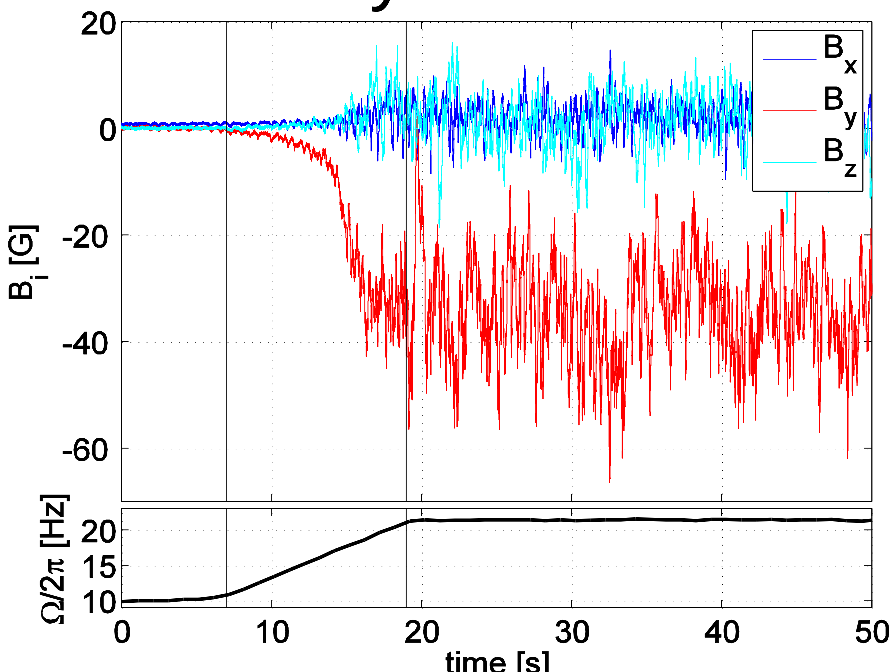



**150L liquid sodium**

**Re=10^6**

**Soft iron disks**

The Dynamo Effect:




In exact counter rotation:
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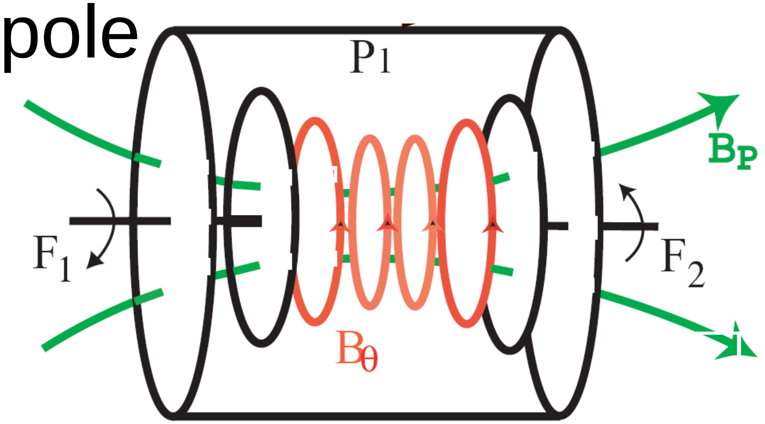
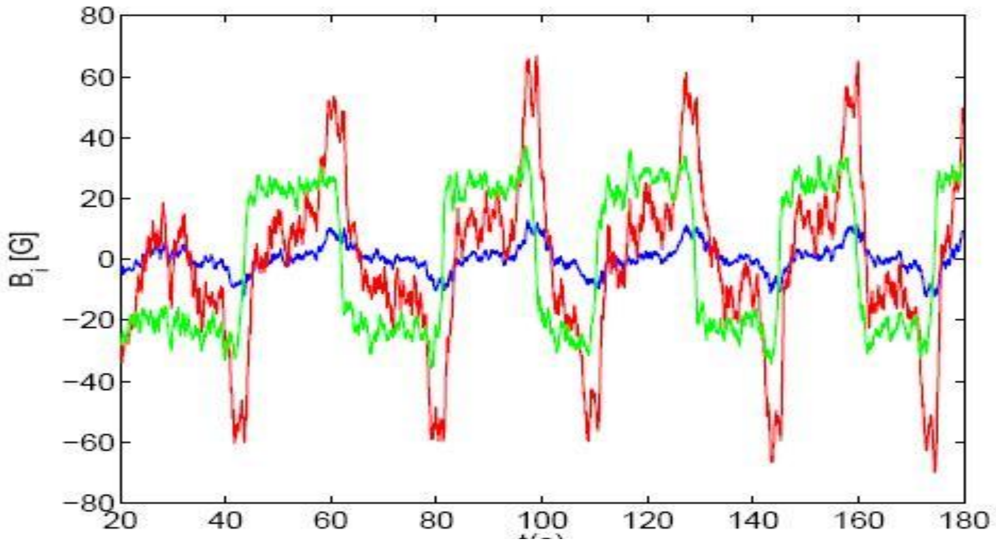
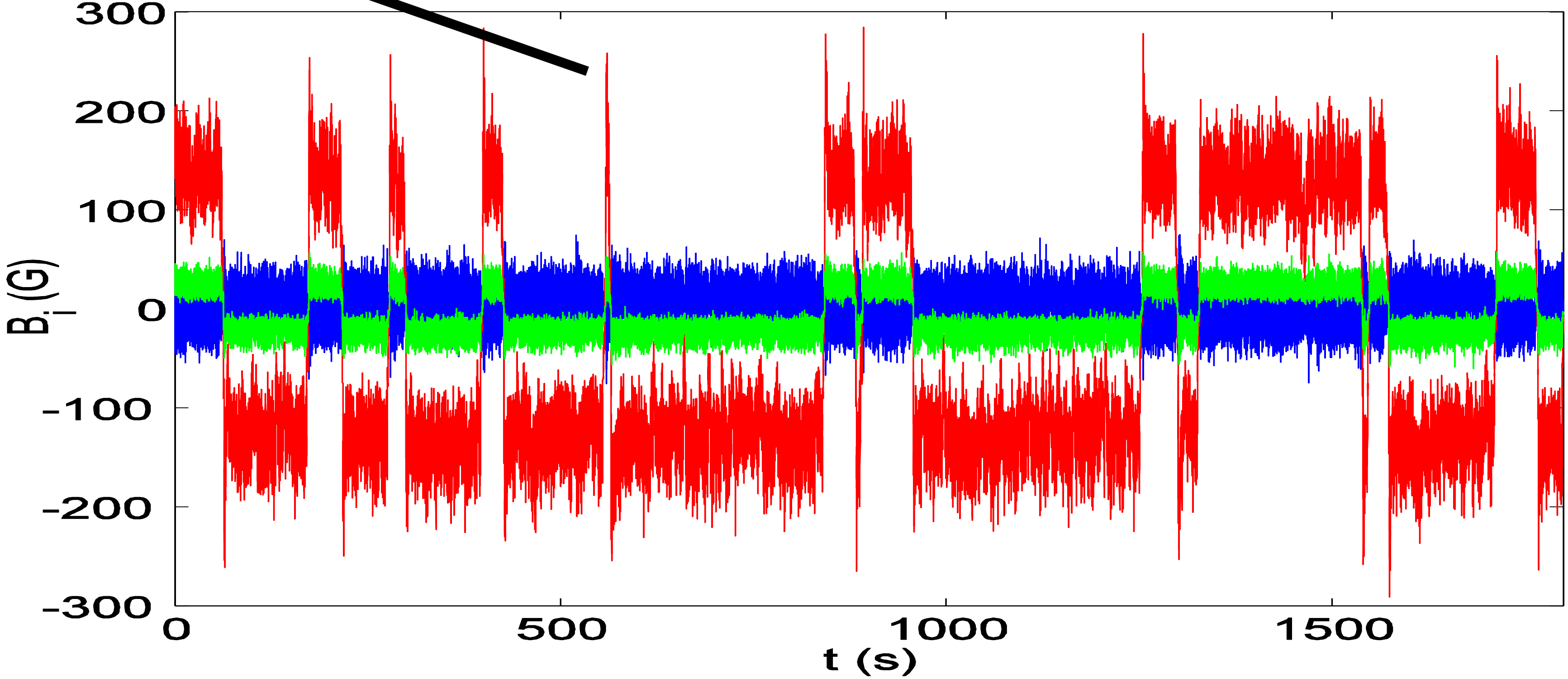


# Disks are rotating at different speeds

## Nonlinear oscillations

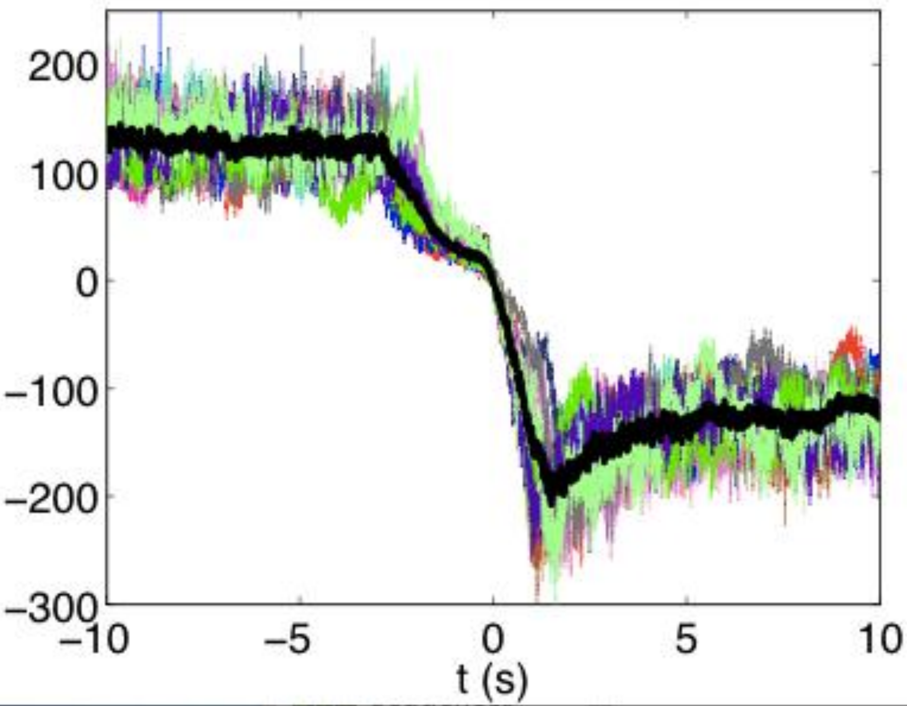


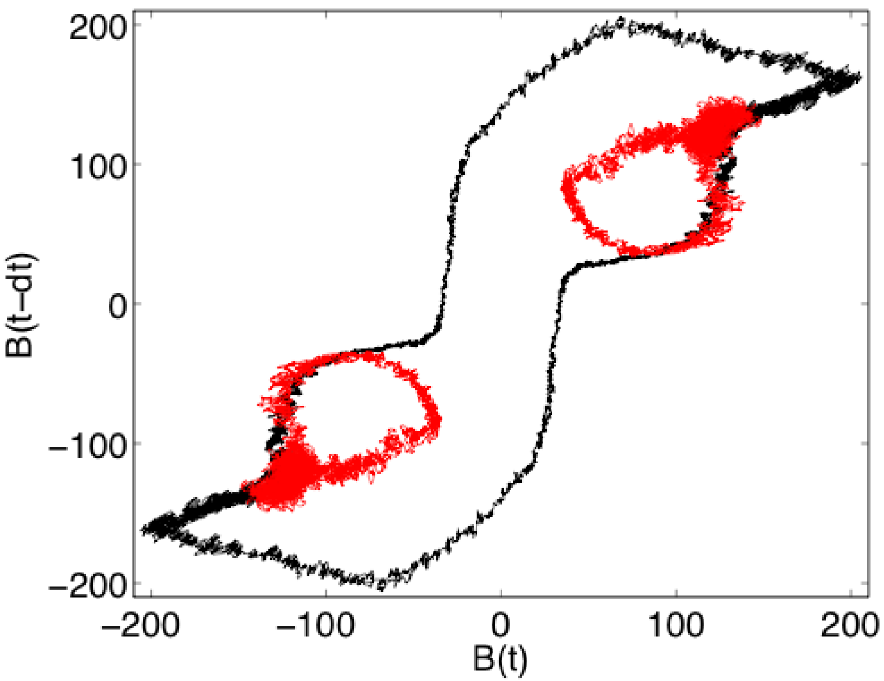

## Reversals



# Robustness of reversals of the magnetic field with respect to turbulent fluctuations

12 superimposed reversals (slow decay, fast recovery, overshoots)

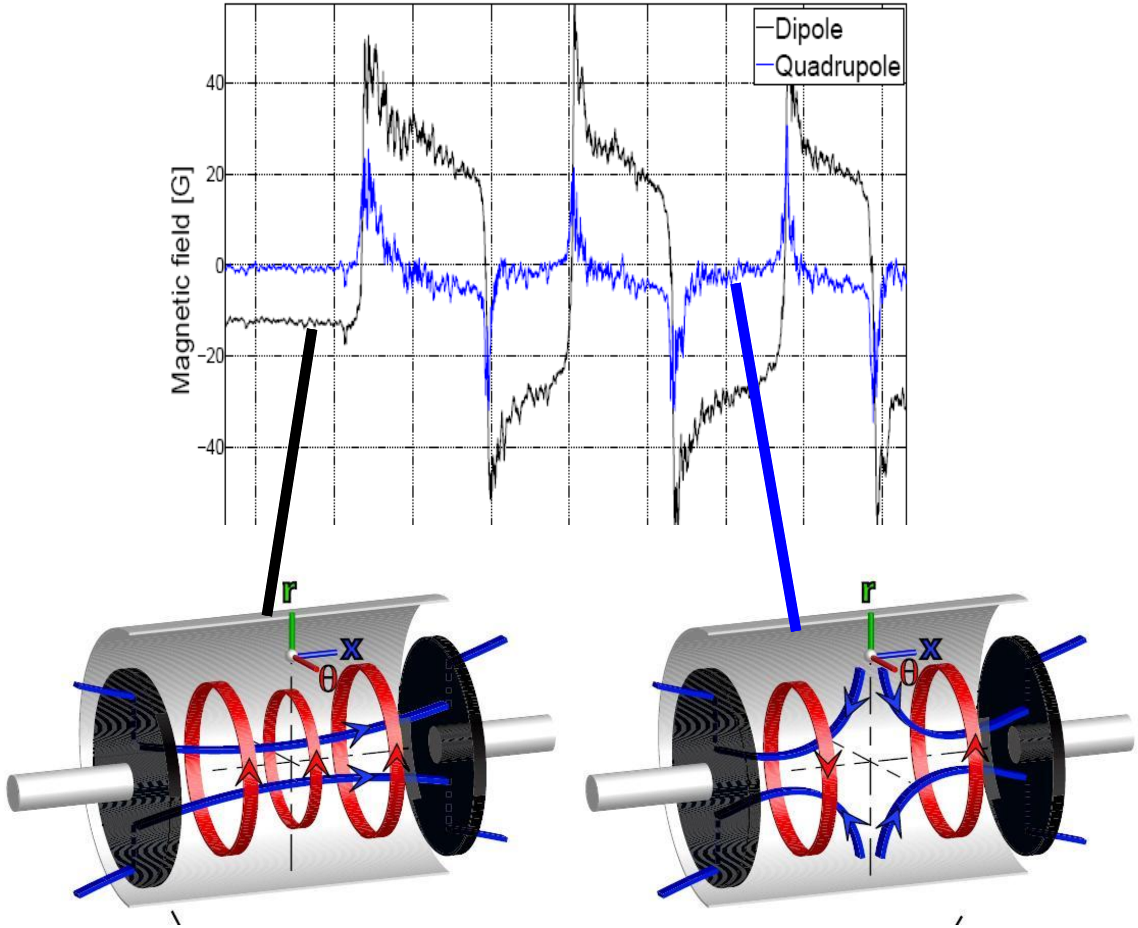



**A low dimensional dynamical system despite high Re ($5.\ 10^6$) ?**

# Dipole and quadrupole decomposition

(C. Gissinger Ph.D Thesis)




Dipole

Quadrupole

All these systems have in common:

- a clear time scale separation between phases of given polarity and the duration of a reversal

- robust trajectories during reversals.

Despite huge Reynolds number (f.i. 10^6 in VKS), turbulent fluctuations do not smear out these trajectories

# Low dimensional model of the dynamics of the magnetic field

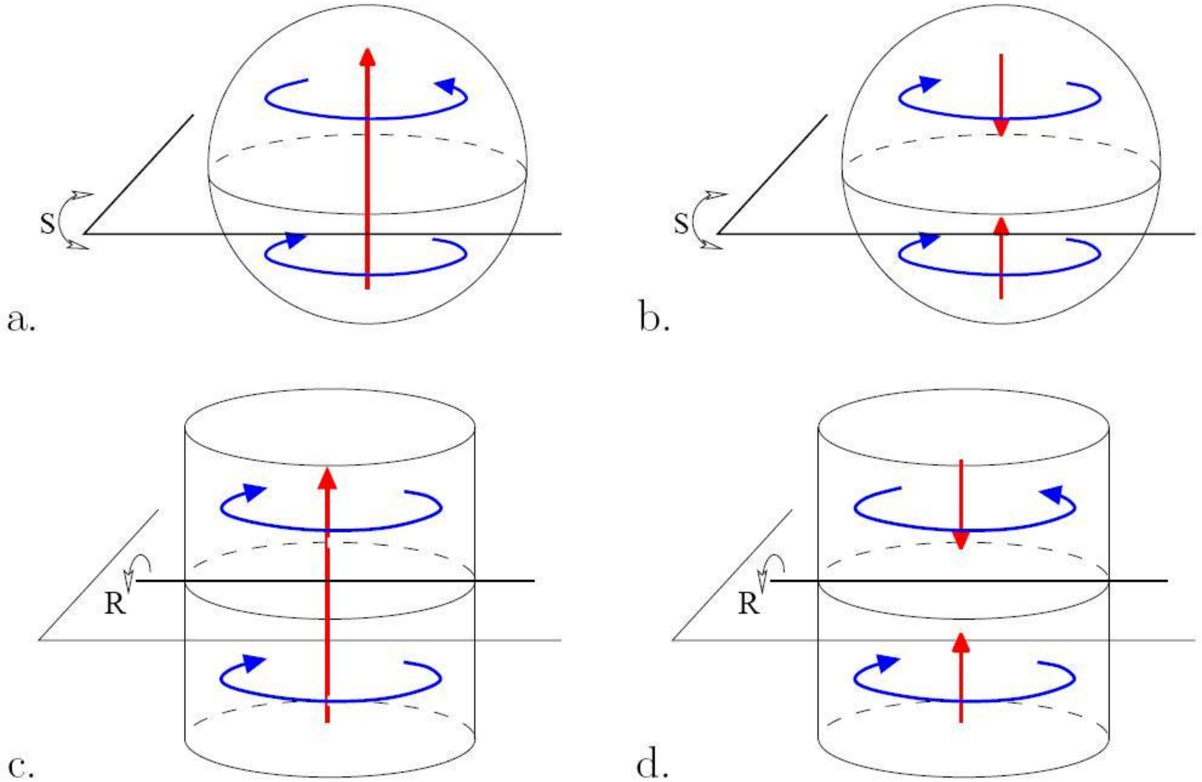
with S. Fauve, E. Dormy (LRA) and J.-P. Valet (IPGP)

Based on symmetry properties of two modes

**Dipole** | **Quadrupole**

**Astrophysical object (The Earth)**

**VKS**



# Equation for dipole and quadrupole

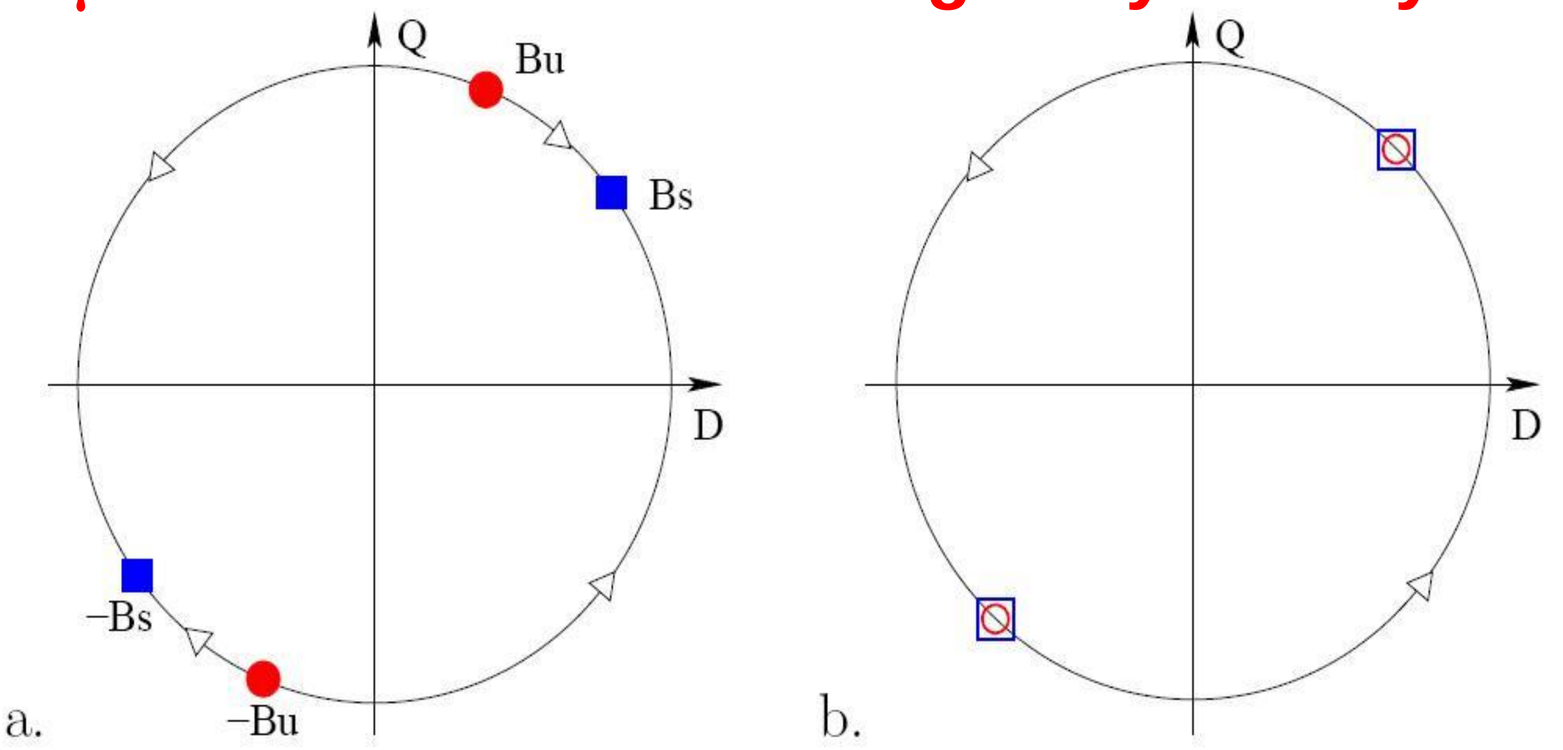
$$\mathbf{B}(r,t) = d(t)\mathbf{D}(r) + q(t)\mathbf{Q}(r)$$

**We set A=d+i q,** $\dot{A} = \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3$

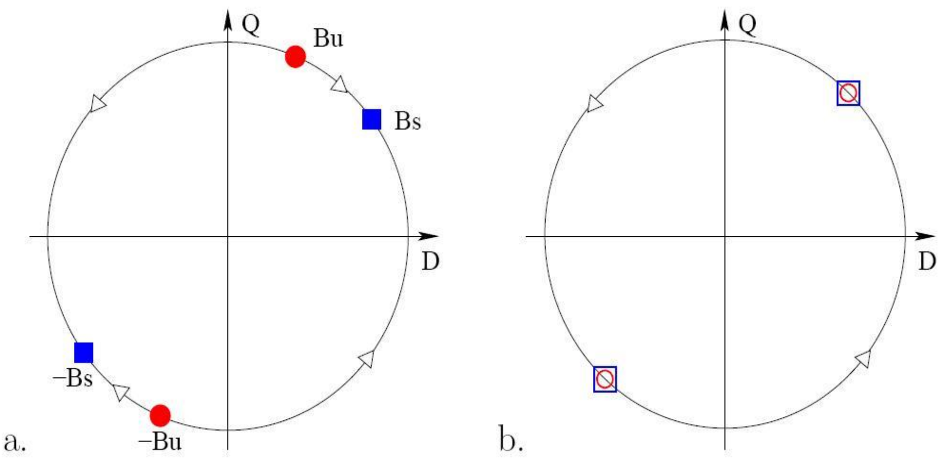
**Phase equation** $A = r \exp(i\theta)$

**Simplified expression** $\dot{\theta} = \mu_i - \nu_r \sin(2\theta)$

**μi measures the breaking of symmetry**



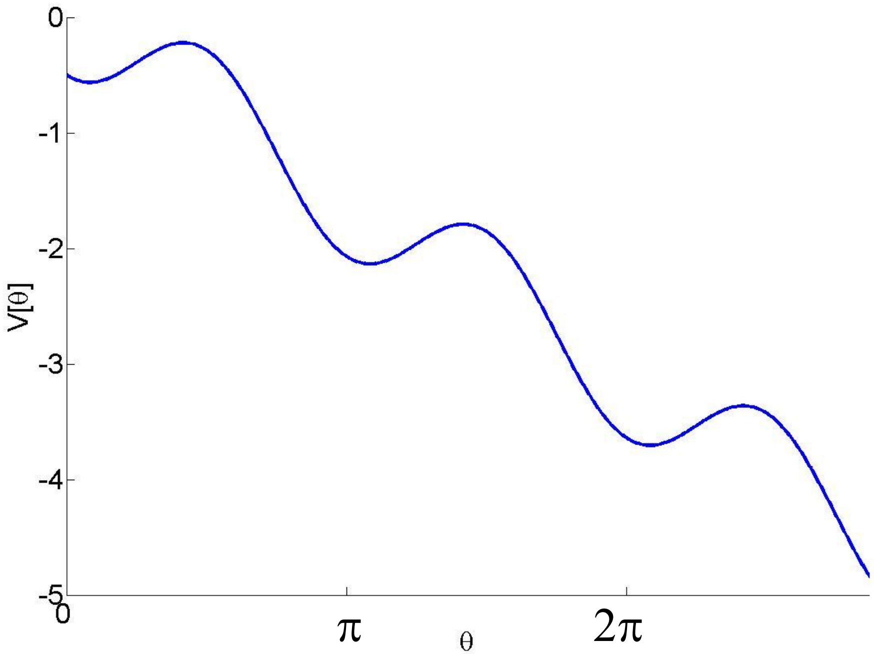
$$\dot{\theta} = \mu_i - \nu_r \sin(2\theta)$$

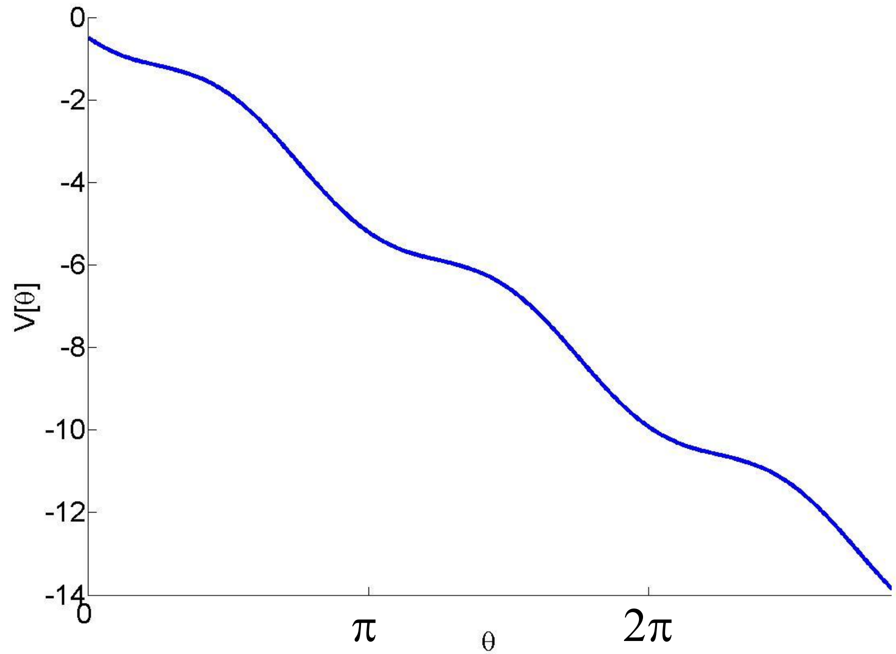



Motion in a potential

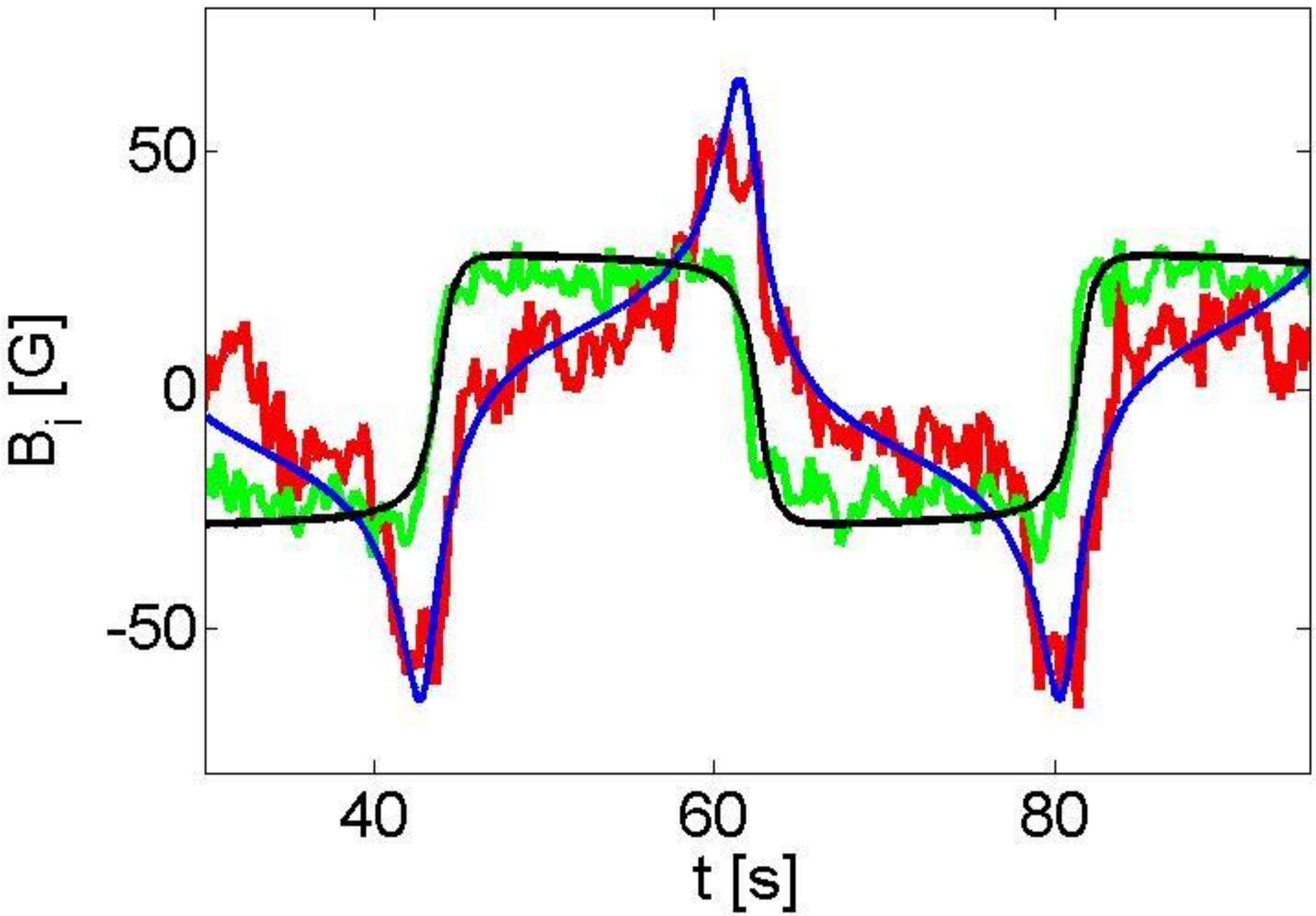
$$\dot{\theta} = -\frac{\partial V}{\partial \theta}$$

$$V[\theta] = -\mu_i \theta - \nu_r \cos(2\theta)/2$$

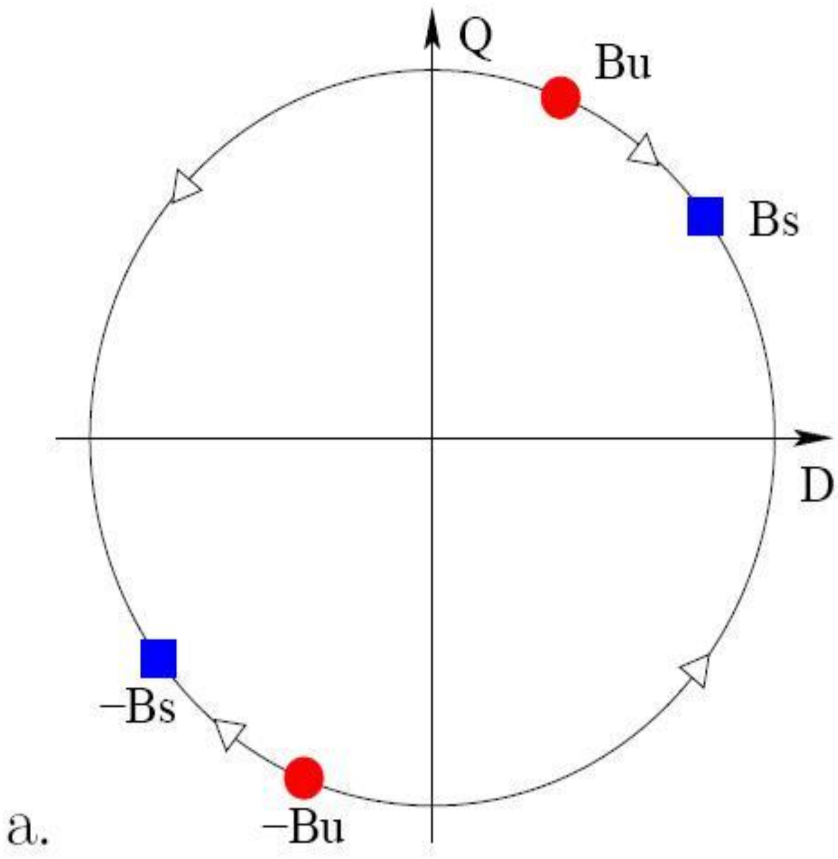


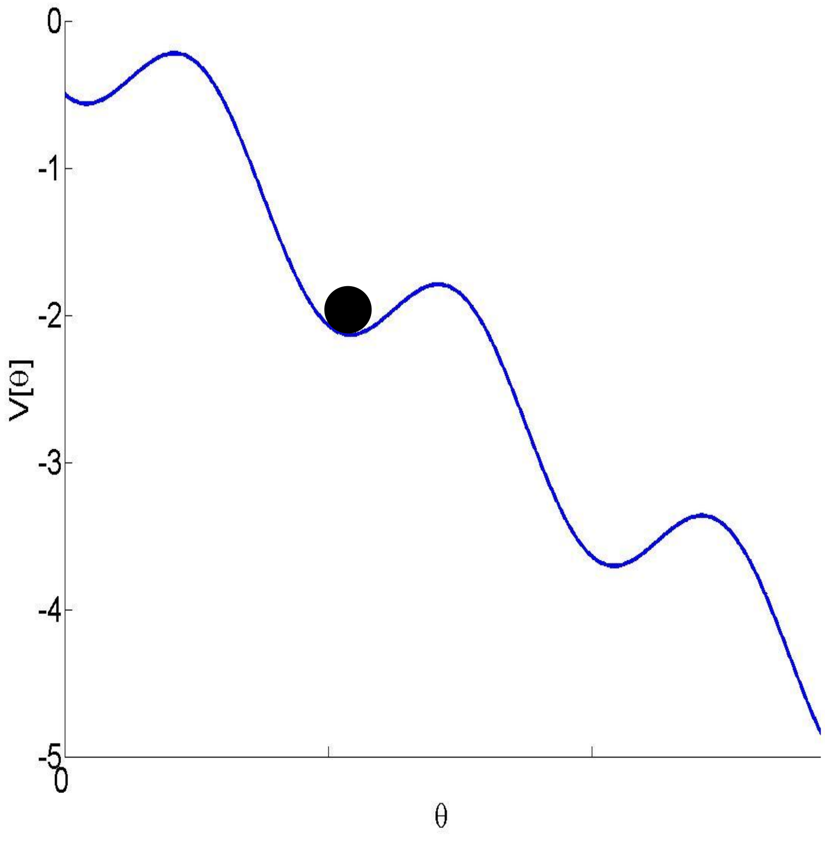



# Comparison between normal form and experiment



# Effect of turbulent fluctuations: a simple mechanism for reversals



# Predictions (for geophysicisists)

**Mechanism and shape of reversals:**

- Two modes of magnetic field are close to a saddle-node bifurcation

- Slow phase followed by a fast phase

**Origine and shape of excursions:**

- Aborted reversals

- Initial phase similar to reversals, ends up without overshoot

**Comparison with the normal form**

$\dot{\theta} = \alpha_0 + \alpha_1 \sin(2\theta) + \Delta\zeta(t)$ **and** $D = R\cos(\theta + \theta_0)$

# Predictions (for this conference only)

**Reversals have all the same shape as a result of large deviation theory.**

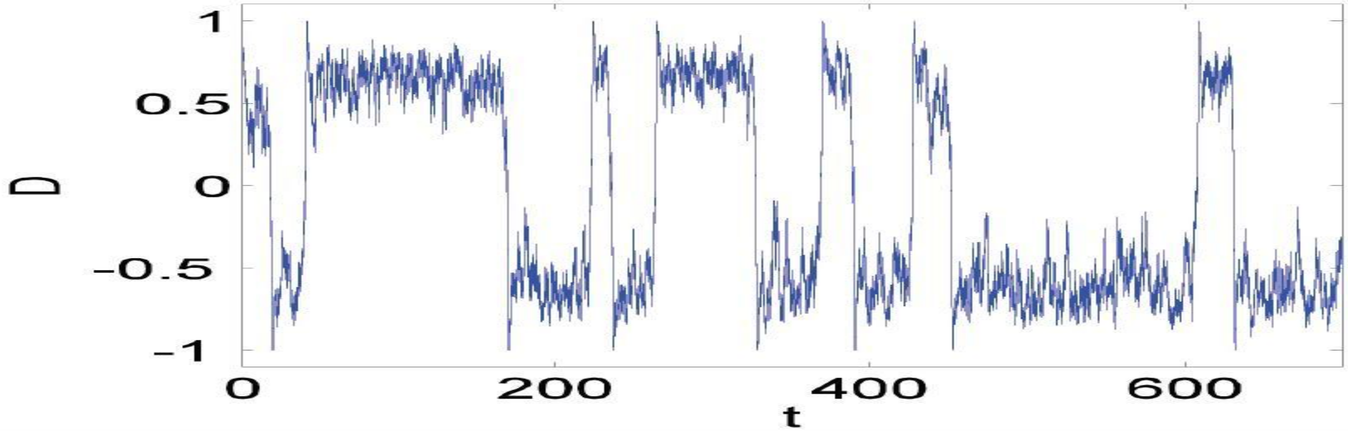
**An exemple of measure concentration for rare events in the low noise limit**

**(Freidlin-Wentzell theory)**

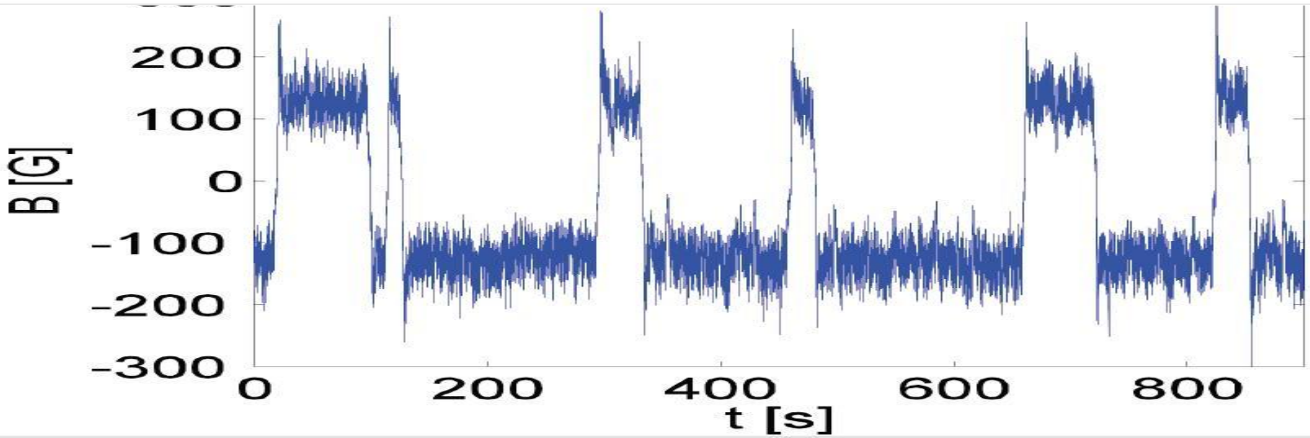
**Comparison with the normal form**

$\dot{\theta} = \alpha_0 + \alpha_1 \sin(2\theta) + \Delta\zeta(t)$ **and** $D = R\cos(\theta + \theta_0)$

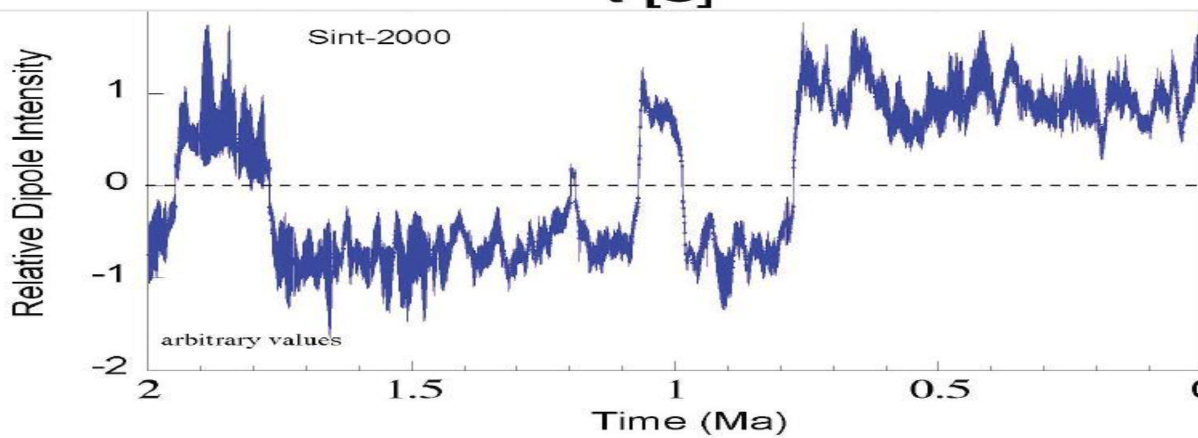
Model




VKS




Earth Dipole

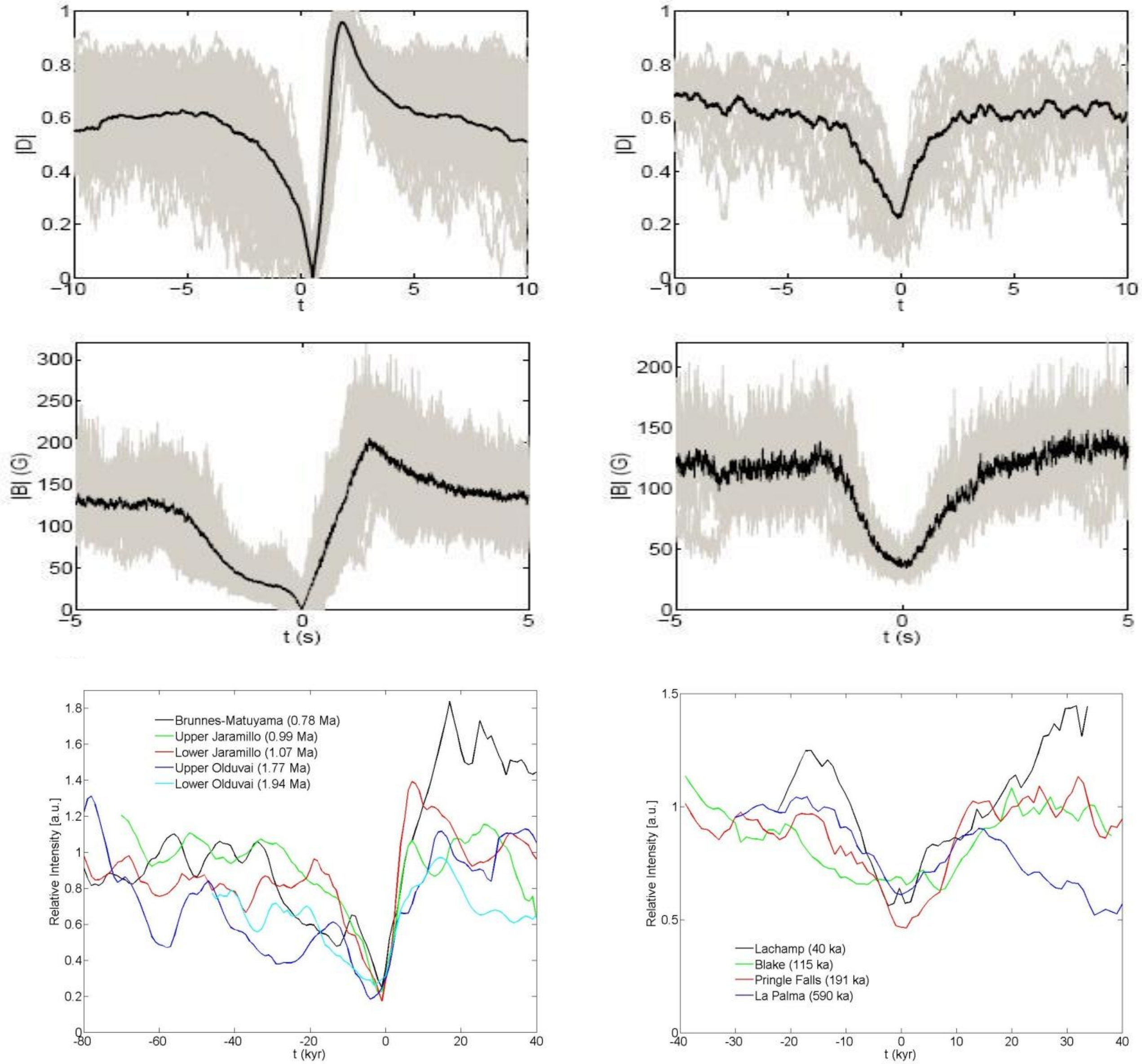


# Reversals | Excursions

## Model

## VKS

## Earth Dipole




a.

b.

c.

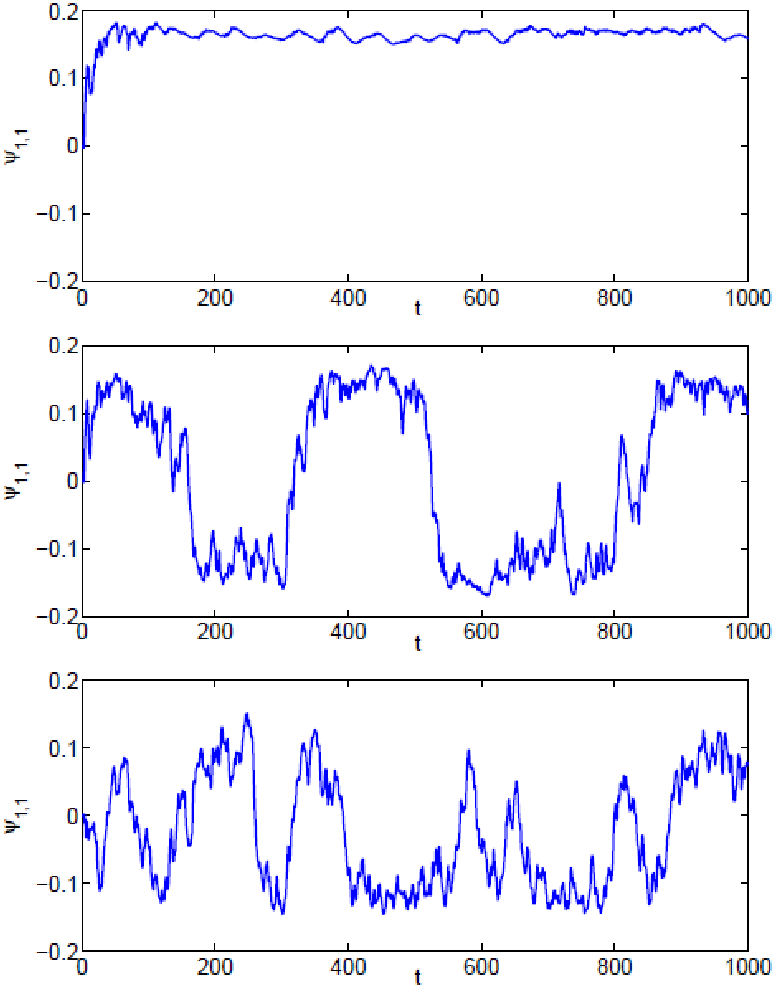
Does « reproducibility of reversal trajectories » imply that the reversals are rare events of a stochastic process?

# Back to Kolmogorov flow
# (B. Gallet Ph. D. Thesis, J. Herault)

DNS of a Kolmogorov flow: reversals of large scale circulation

A few large scale modes dominate:

$$\begin{aligned}\dot{D} &= -\nu D - Q_x Q_y \\ \dot{Q}_x &= +Q_x - Q_y D - Q_x^3 \\ \dot{Q}_y &= \mu Q_y + D Q_x\end{aligned}$$



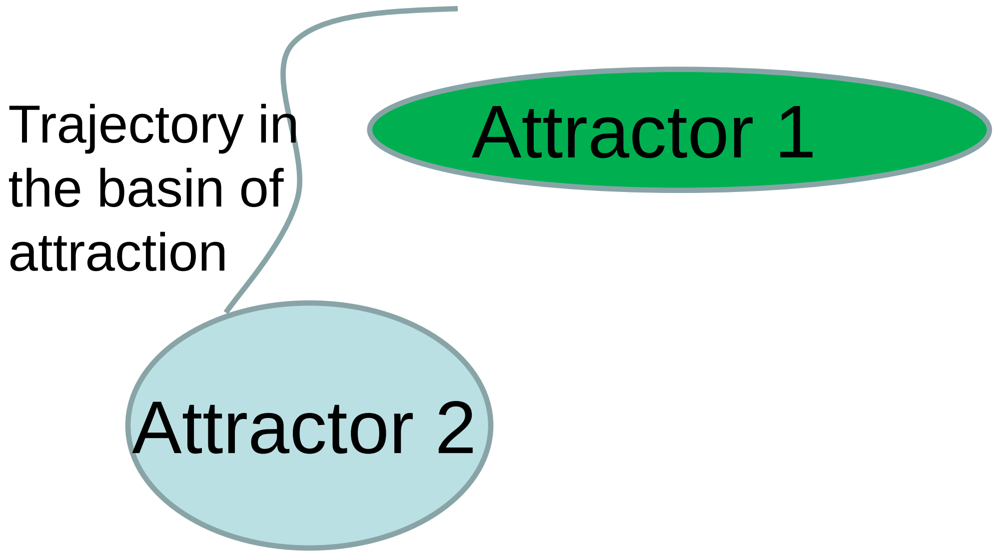
## Numerical simulations for the low dimensional model (purely deterministic) show:

- The reversals take place below a certain value of the control parameter

- Above the threshold the system is chaotic

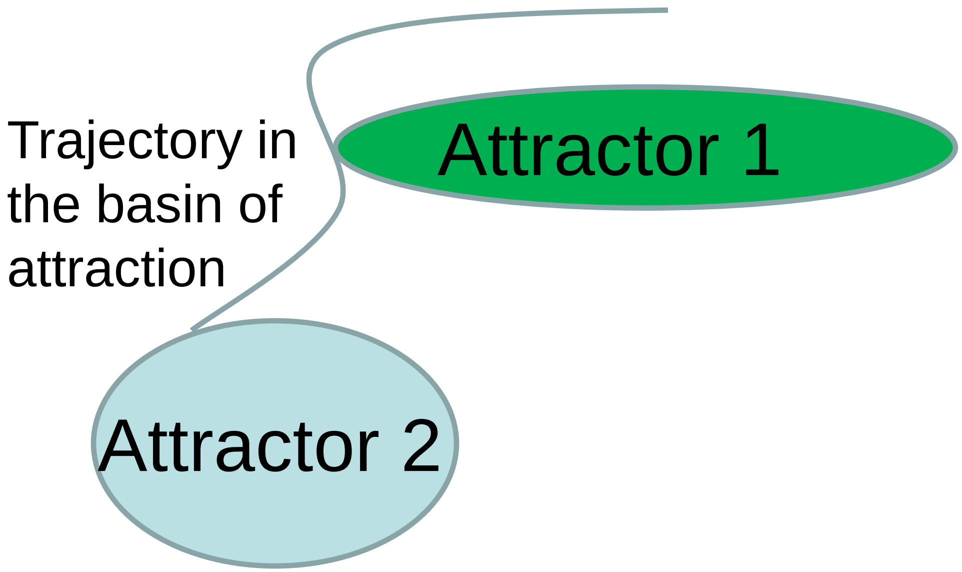
- Slightly below the threshold, reversals have the same shape

The reversals are generated by a crisis mechanism (Grebogi et al. PRL 1982):



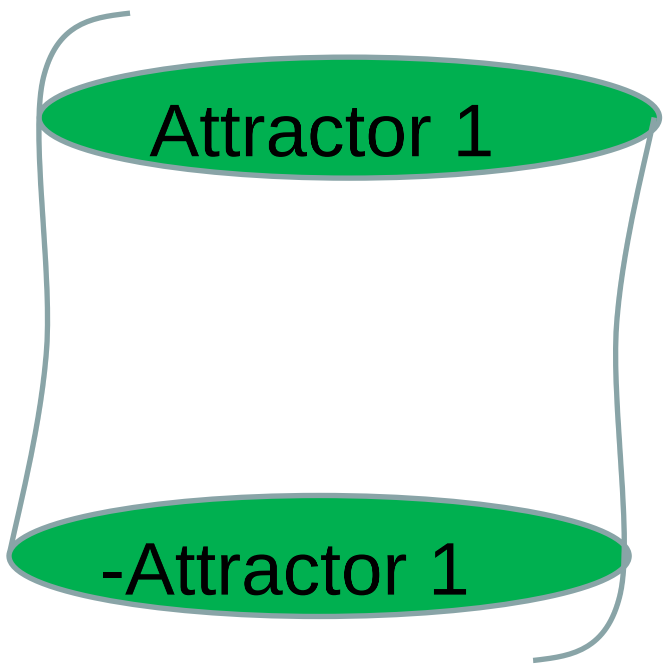

A chaotic attractor collides with the basin of attraction of another attractor. Trajectories escape from the first attractor.

The reversals are generated by a crisis mechanism (Grebogi et al. PRL 1982):

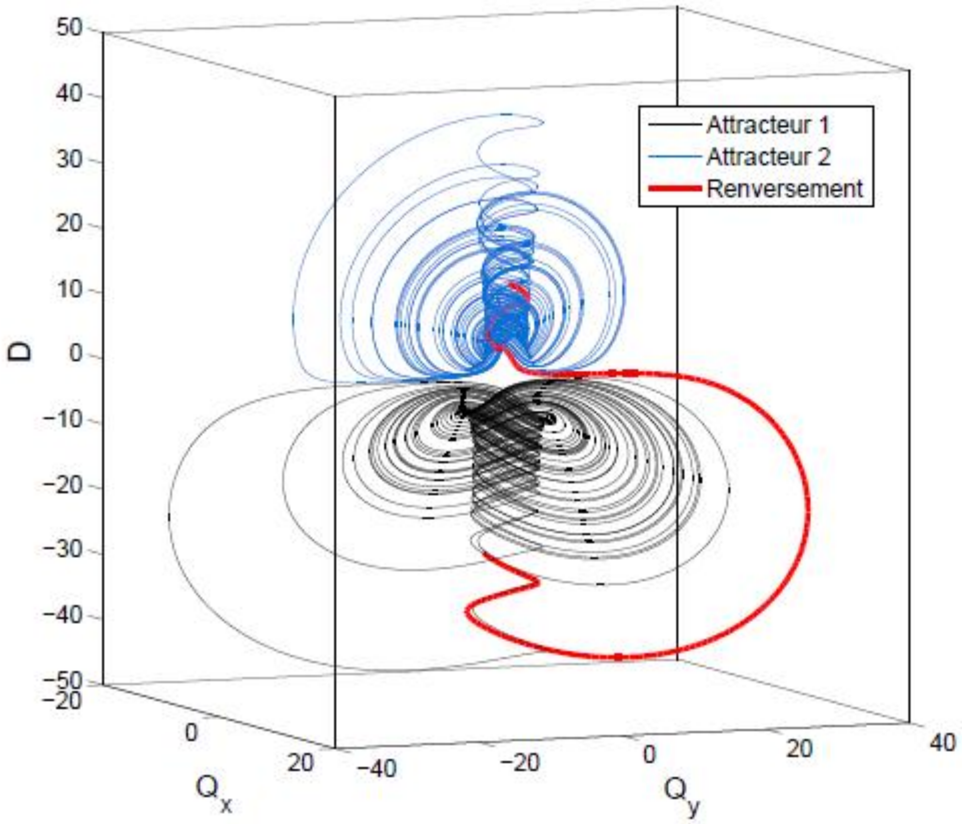



A chaotic attractor collides with the basin of attraction of another attractor. Trajectories escape from the first attractor.

Because of the symmetries of the problem,
the second attractor is the opposite of the first one and successive escapes are reversals




Connecting trajectories

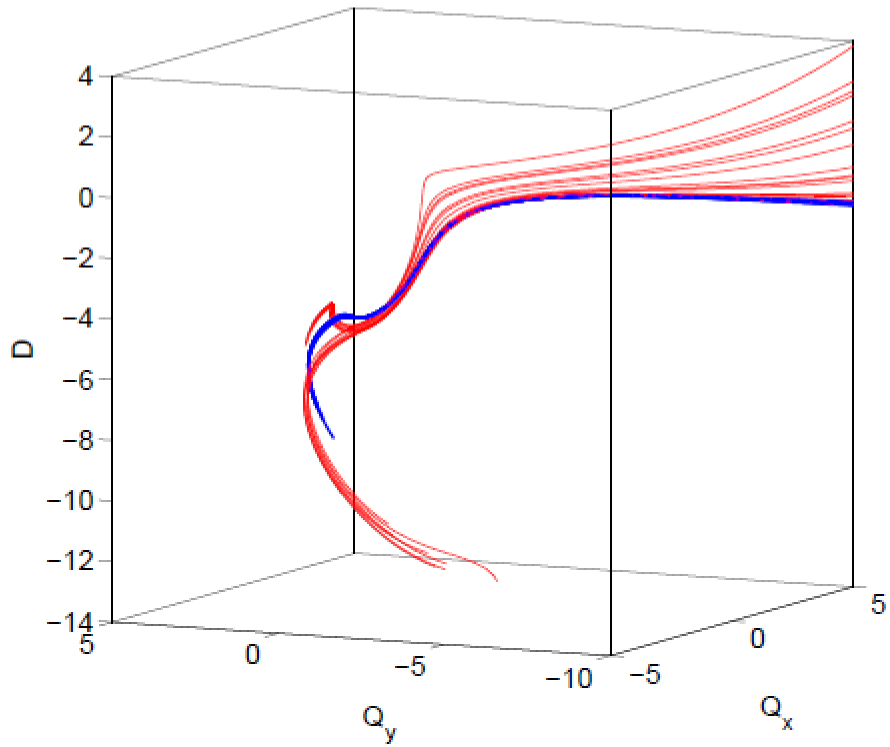



Phase space in the low dimensional model:
Red trajectories connects the blue and black attractors

(see also C. Gissinger EPJ B 2012)

Trajectories are concentrated in phase space:
time series of different reversals are the same.

Because reversals are trajectories that starts on a very small domain in phase space




Blue: close to threshold
Red: far from threshold

# Conclusion

Variety of systems (Dynamo, R-B convection, Kolmogorov flow…), large scale field displays reversals

Described by different low dimensional models
(randomness from stochastic process or low dimensional chaos)

In common:

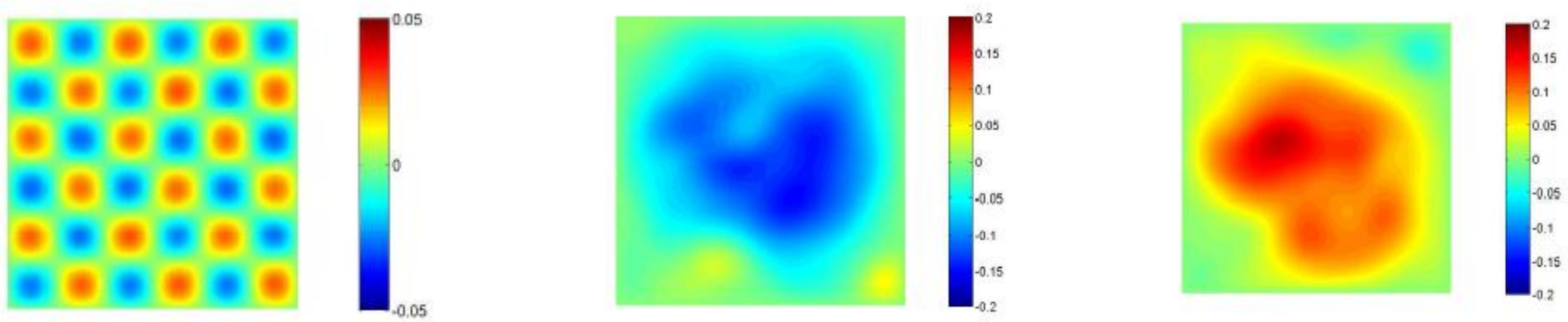
- Existence of two opposite attractors
- fluctuations/wandering in phase-space push the system aways from the basin of attraction of one state and initiate a reversal.

These are unlikely events, and this is responsible for
- the time separation between reversals duration and inter-reversals duration
- the similarities between trajectories

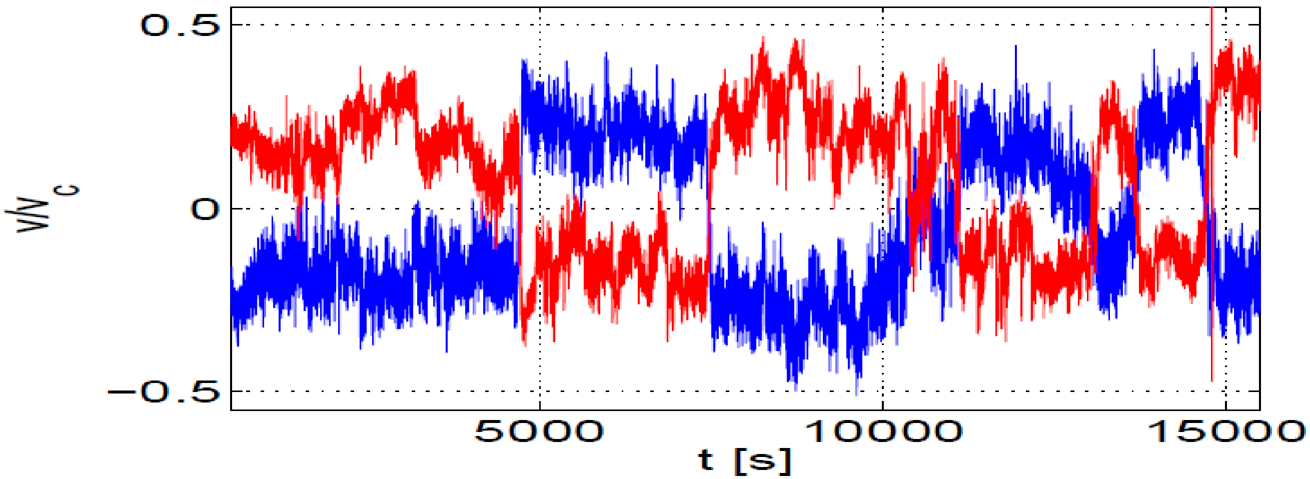
No, robustness of reversal trajectories is not always caused by measure concentration in the low noise limit of a random process

# Large scale circulation in a 2D Kolmogorov flow
(J. Herault, G. Michel, B. Gallet, S. Fauve)

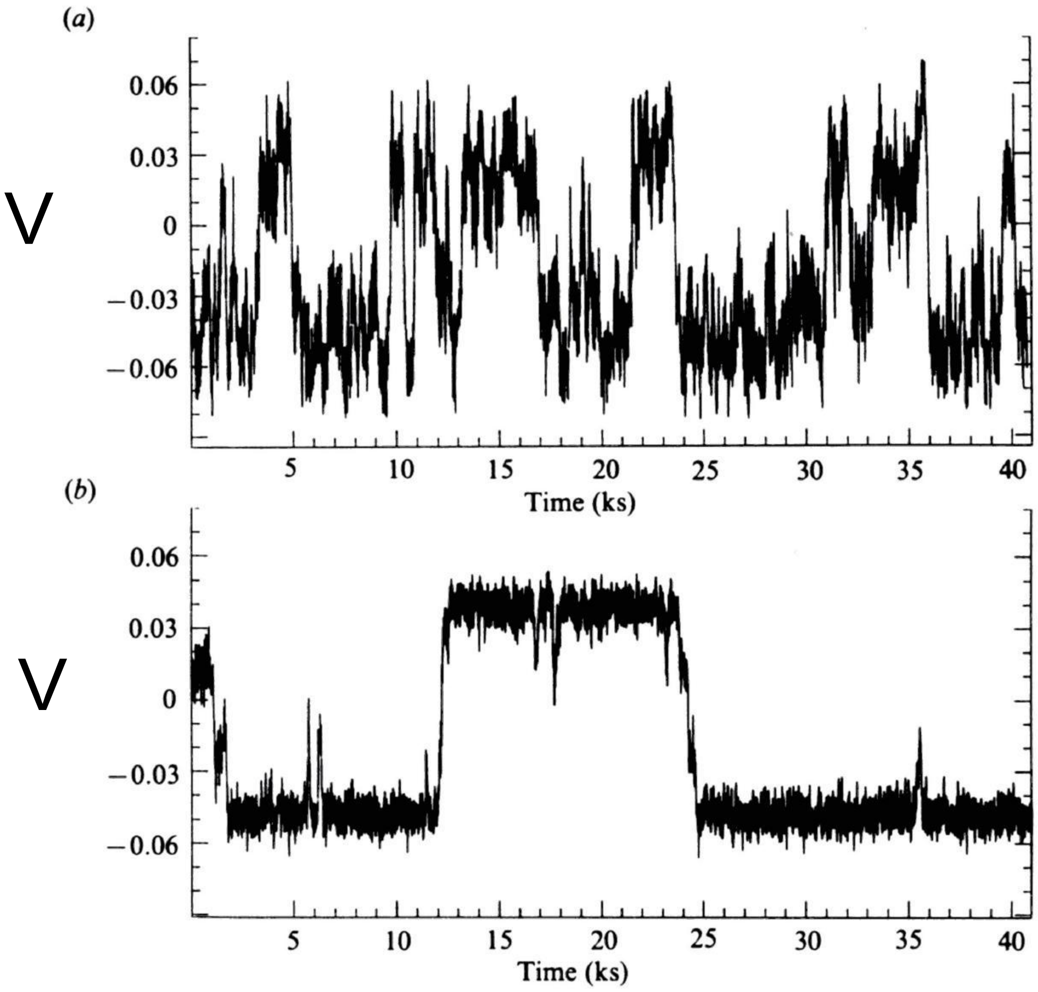
Exp (Sommeria 86): periodic electrical forcing (array of electrodes) in a liquid metal layer plunged into a vertical magnetic field




Forcing drives large scale circulation (2D inverse cascade)



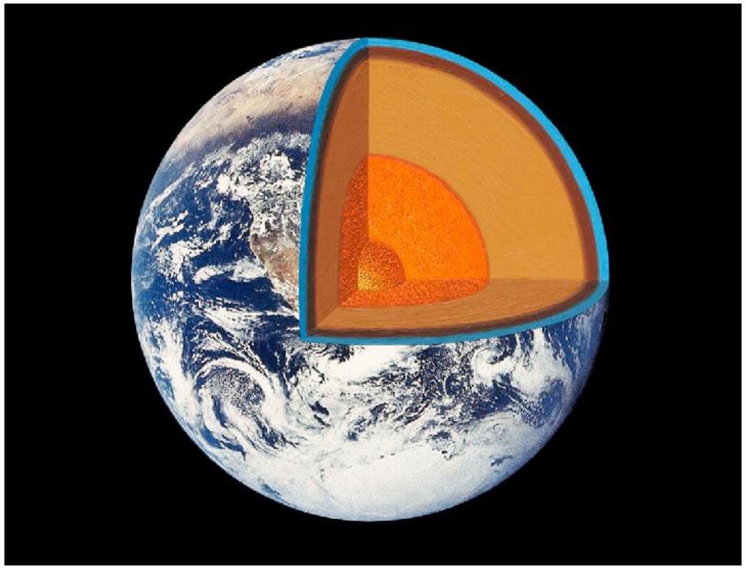
# Reversals of the large scale circulation driven by two-dimensional periodic flows



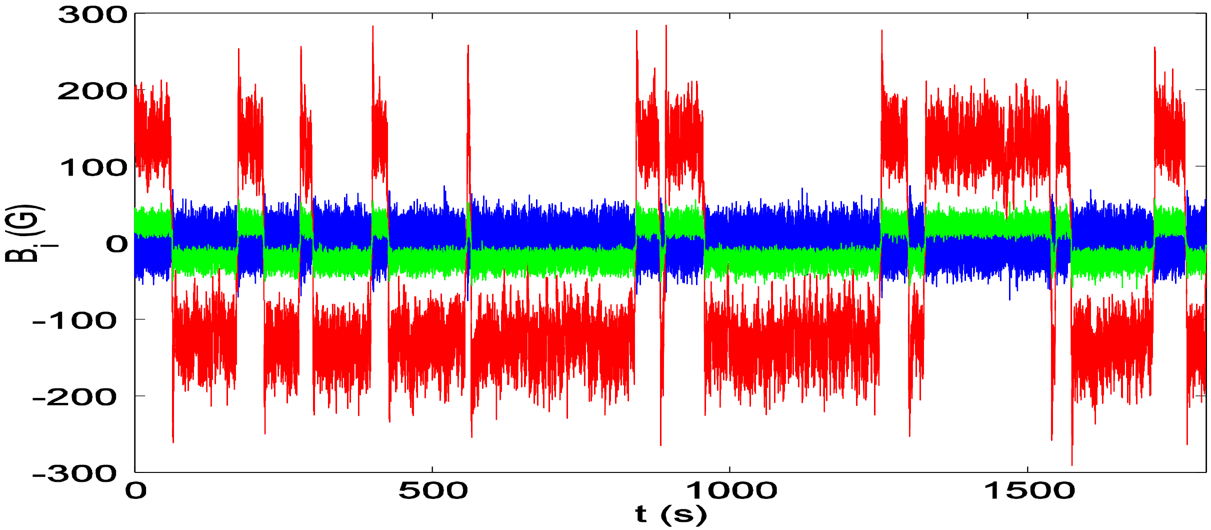

Sommeria, JFM 170 (1986)

# Reversing magnetic fields

The Earth magnetic field



Various DNS and dynamo models

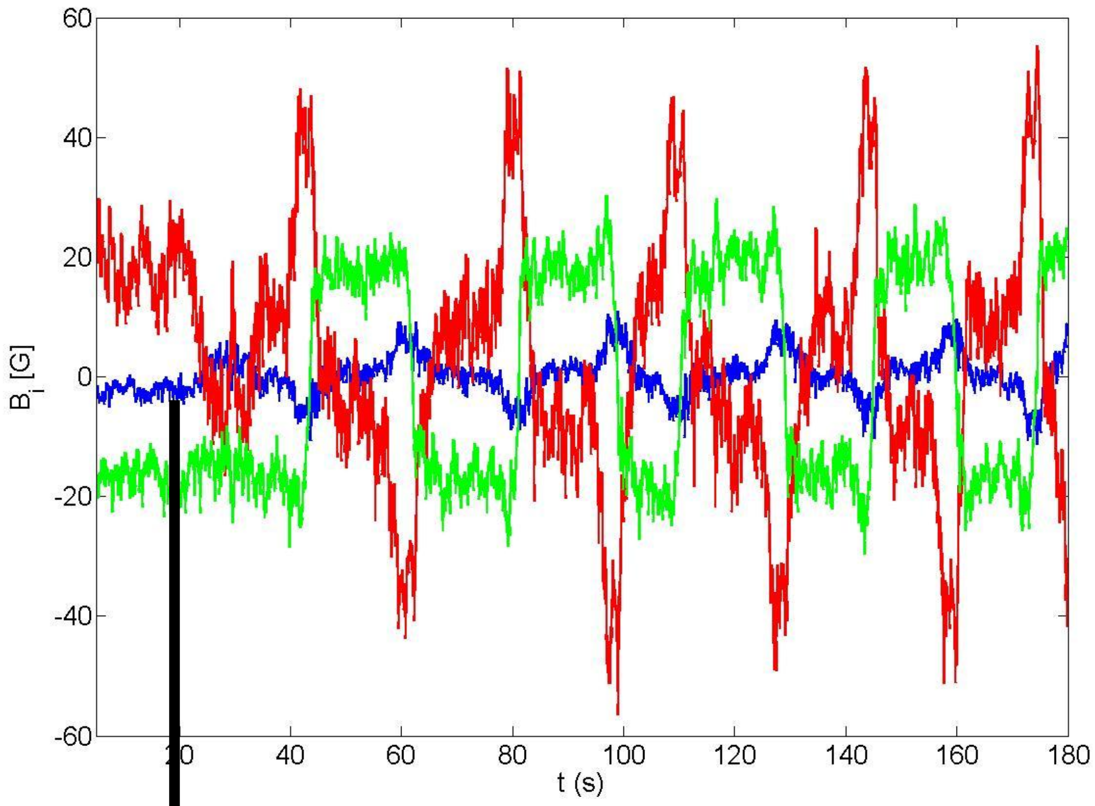



VKS experiment:
Berhanu et al EPL (2007)

**No reversals in exact counter rotation (stationary regime).**

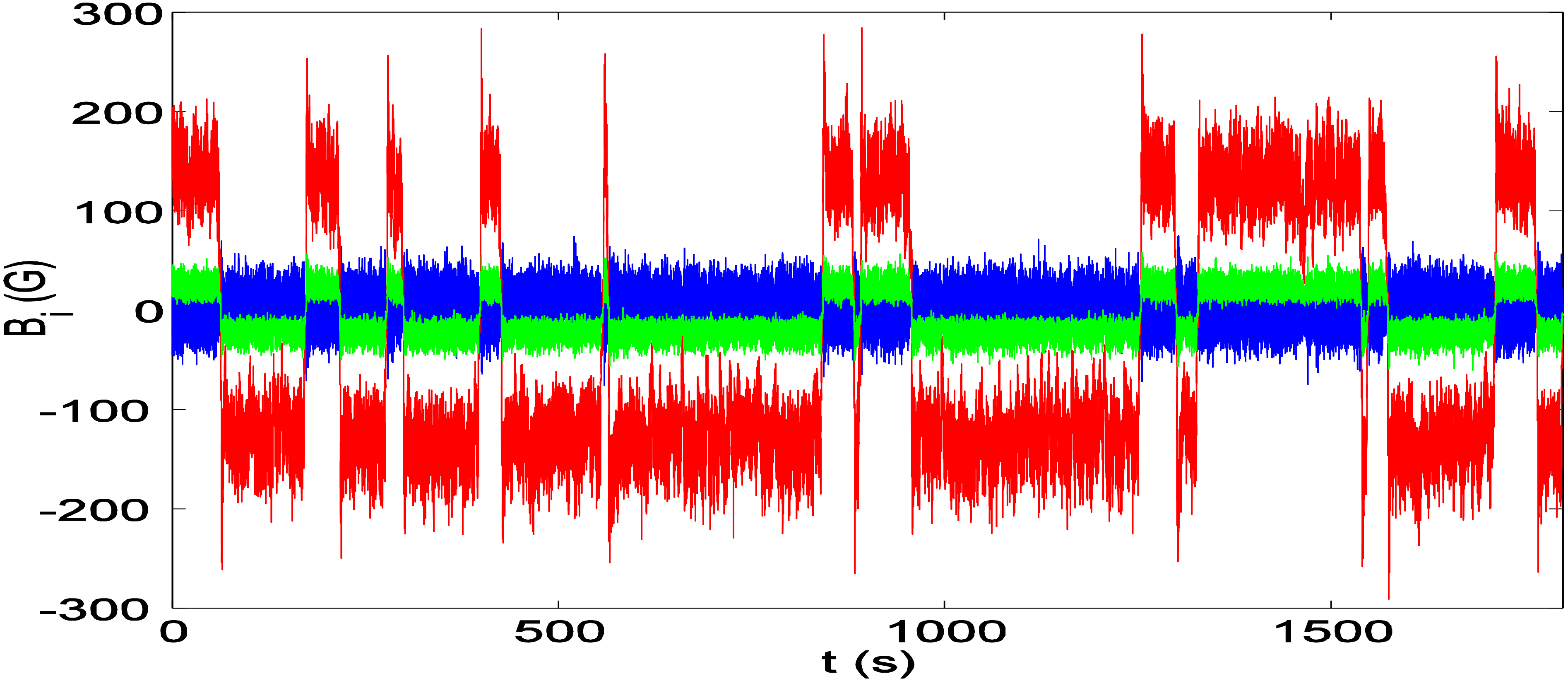
**When disks rotate at different frequencies**

# Nonlinear oscillations



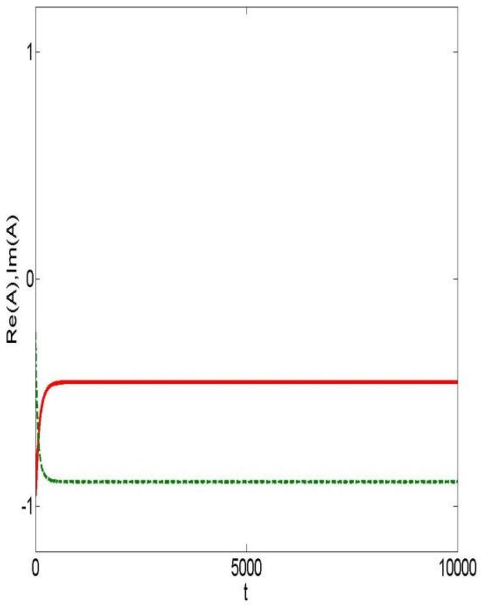

Very small change in disk velocity

# Other example:
# Reversals

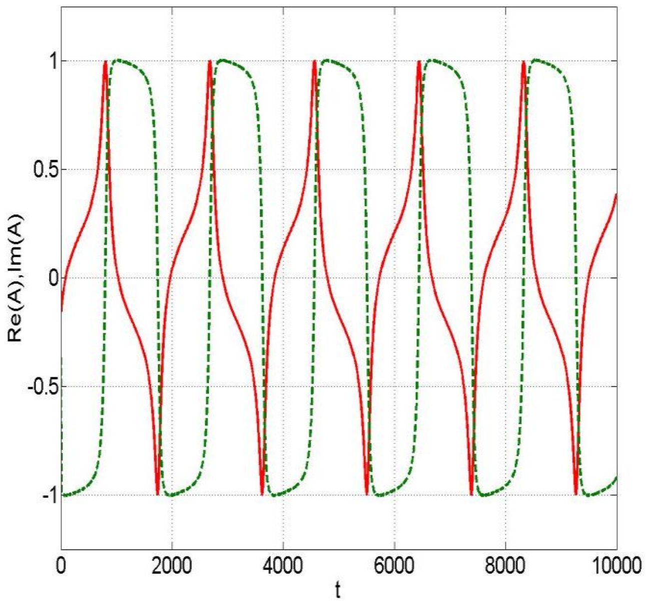


If F1=F2: coefficients are real
coupling cannot drive the saddle-node bifurcation

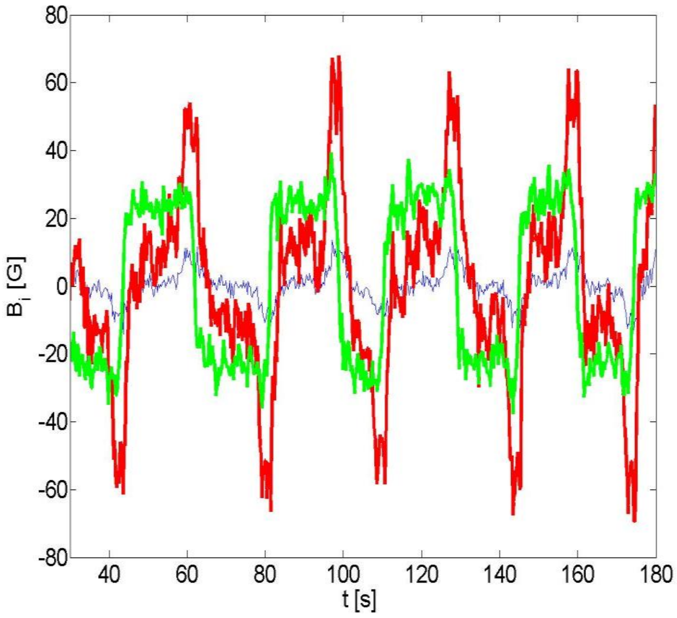
Examples of time-series obtained (coefficients are prescribed functions of f $\alpha$ F1-F2):




f=0.5



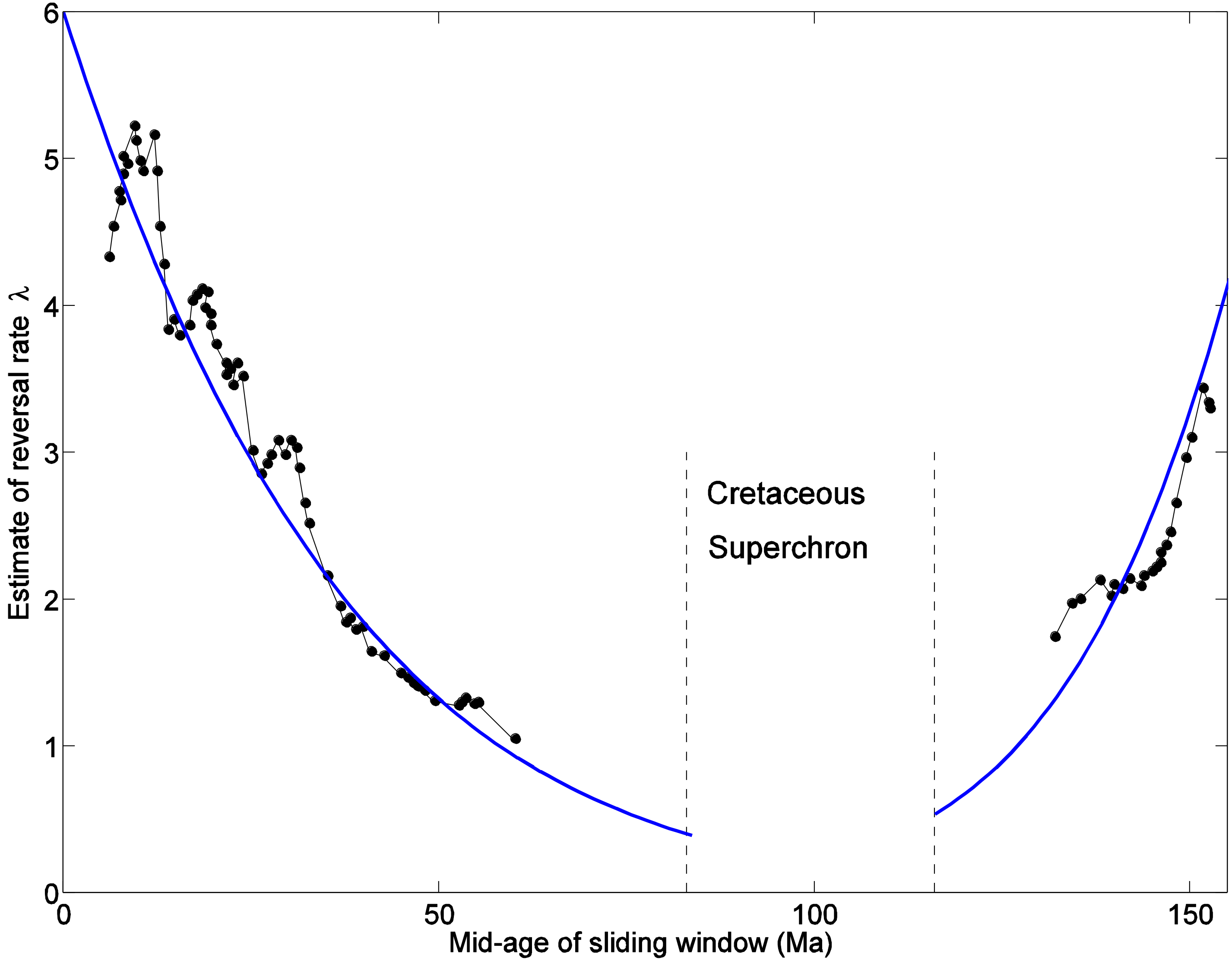

f=1.05




VKS

# Reversal rate:

assume linear in time evolution of the distance to saddle-node onset



# Other reversing systems

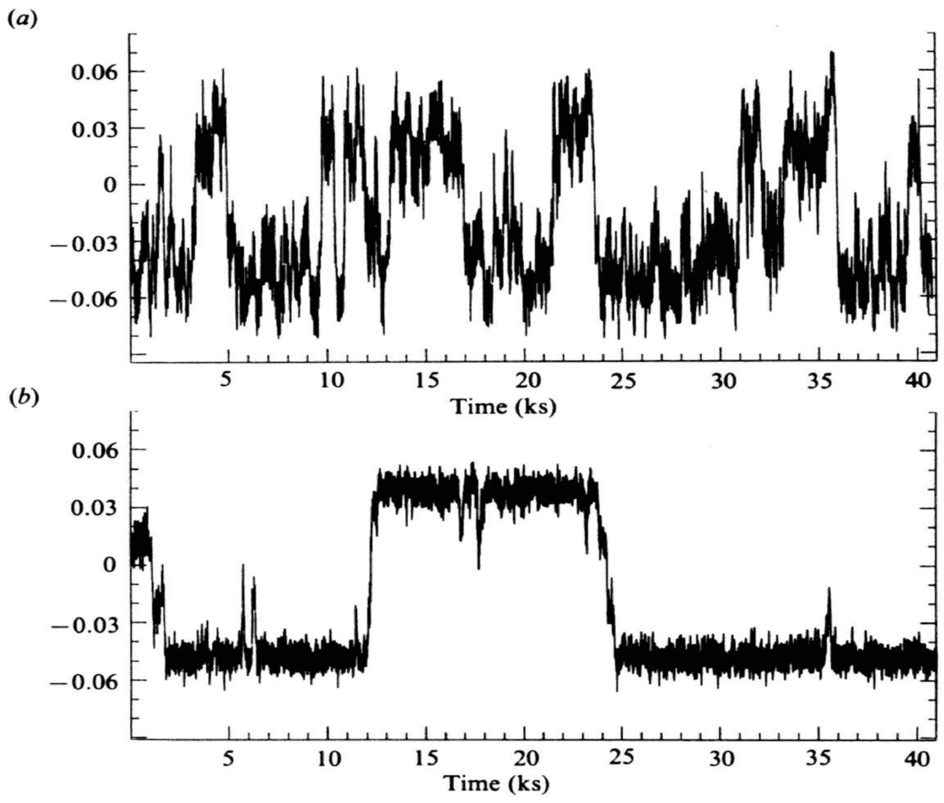
Large scale fields generated on a turbulent background

-Turbulent Rayleigh-Bénard Convection (Krishnamurty et Howard 1982, Liu et Zhang 2008)

-Large scale circulation

driven by two-dimensiona

periodic flows

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JFM 170 (1986)

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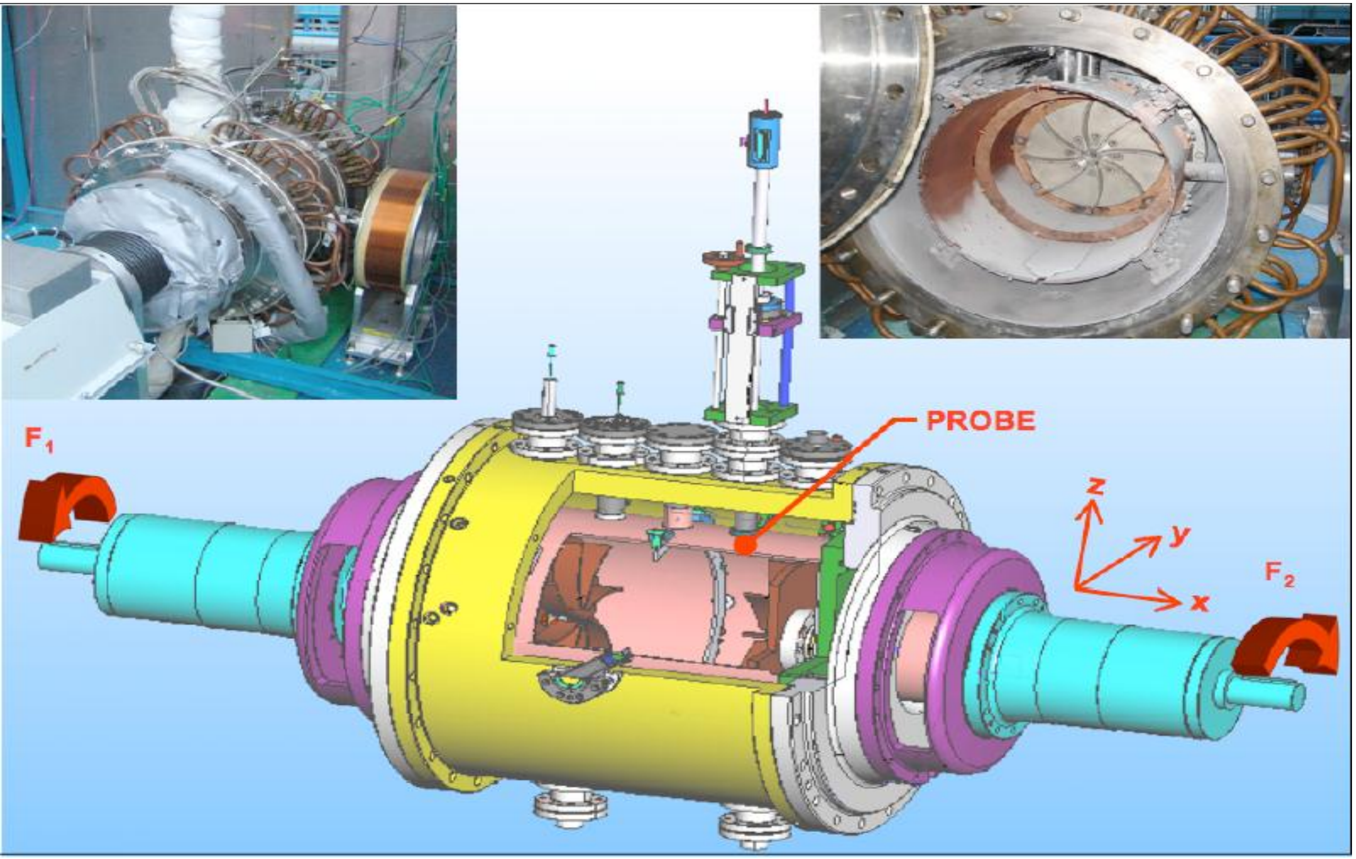
CEA-Saclay **(S. Aumaître, A. Chiffaudel, B. Dubrulle, F. Daviaud, R. Monchaux)**




**2x150 kW motors,**

**Re=10^6**

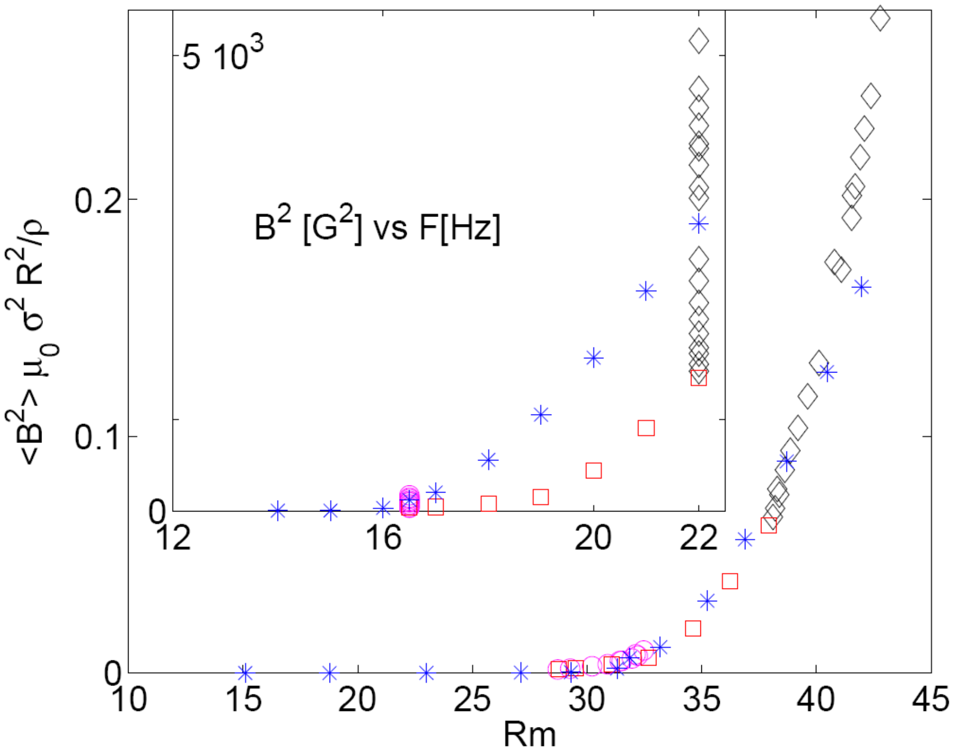
**150L liquid sodium**

**(100-160 C)**

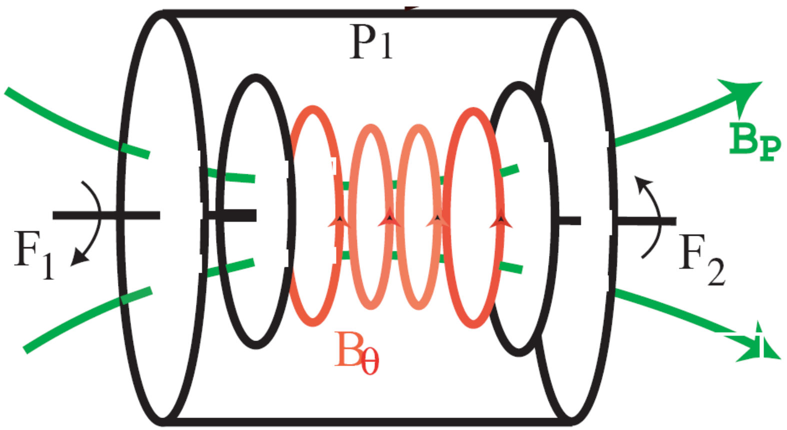
**Soft iron disks**

Magnetic field

at saturation:

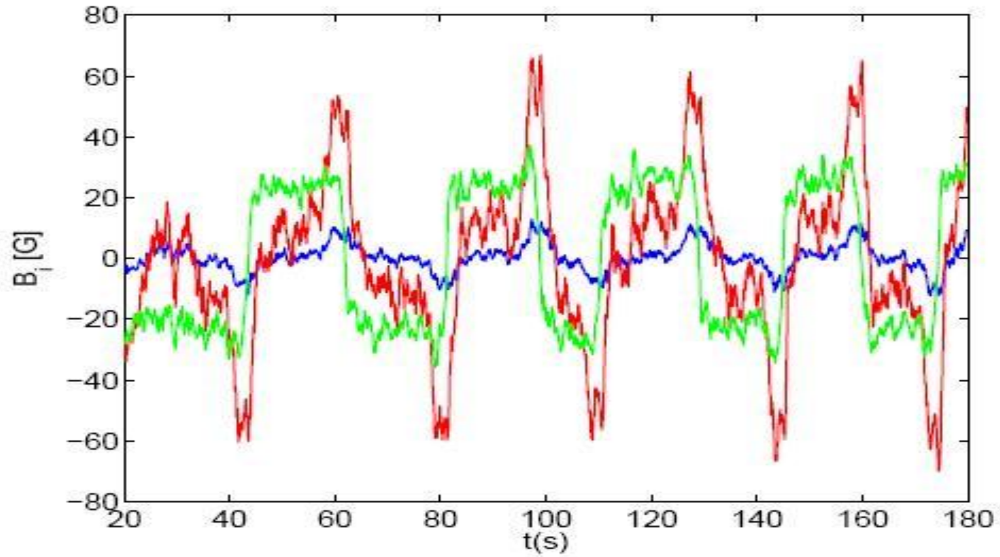



Spatial Structure of B:
an axial dipole

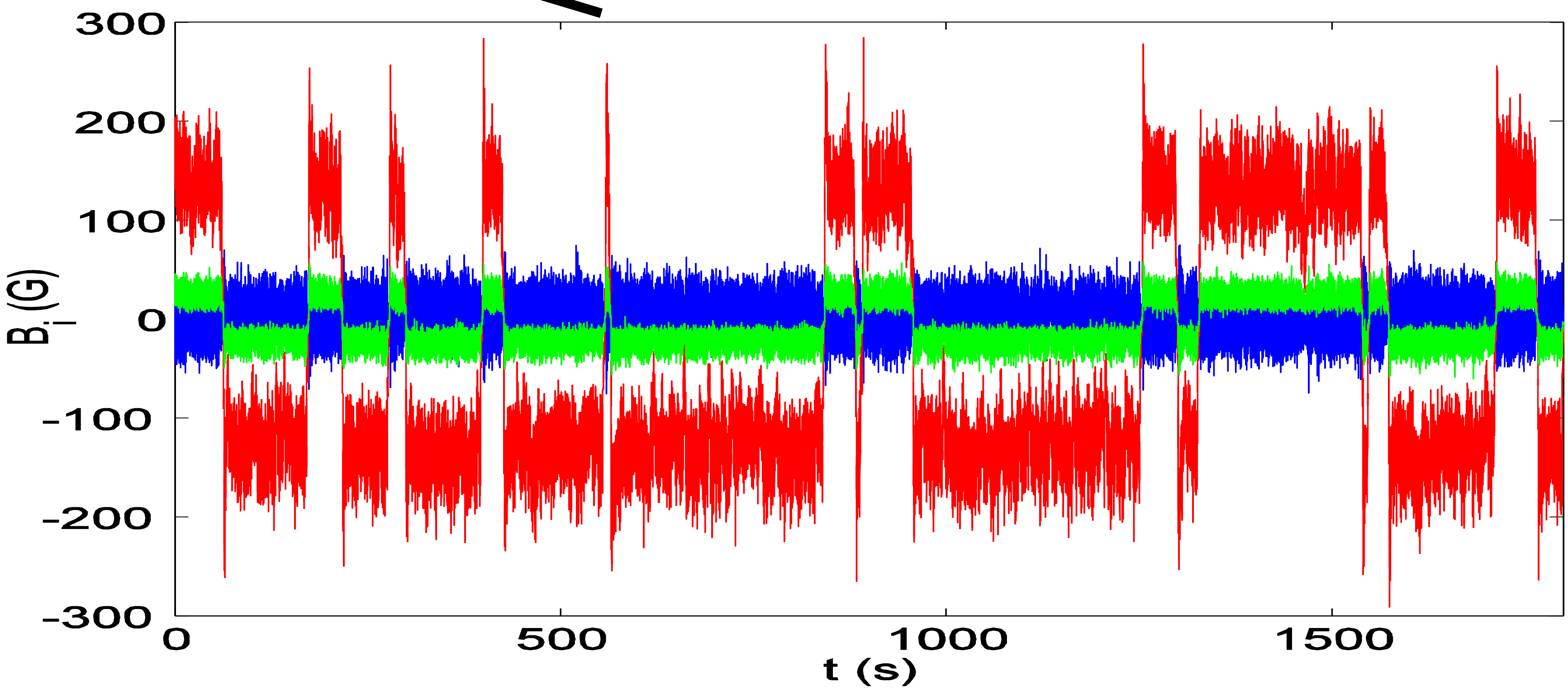


**Disks are rotating at different speeds**

**Nonlinear oscillations**




**Reversals**



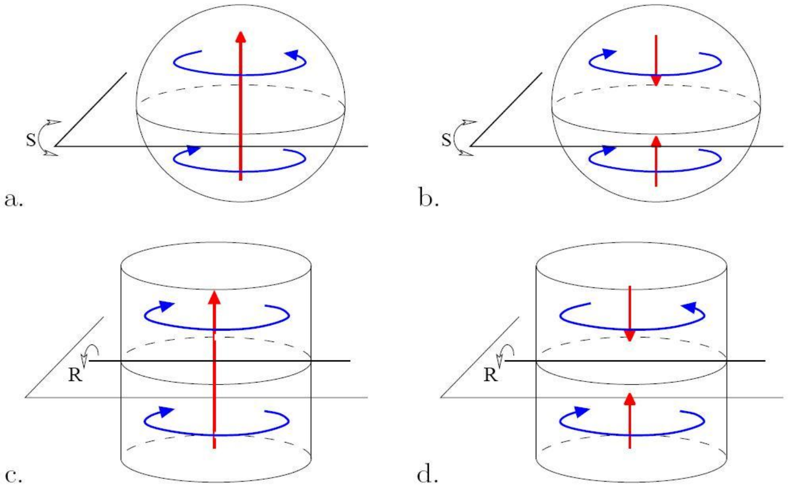
# A mechanism for magnetic field dynamics

Low dimensional dynamics of the magnetic field

Symmetry properties

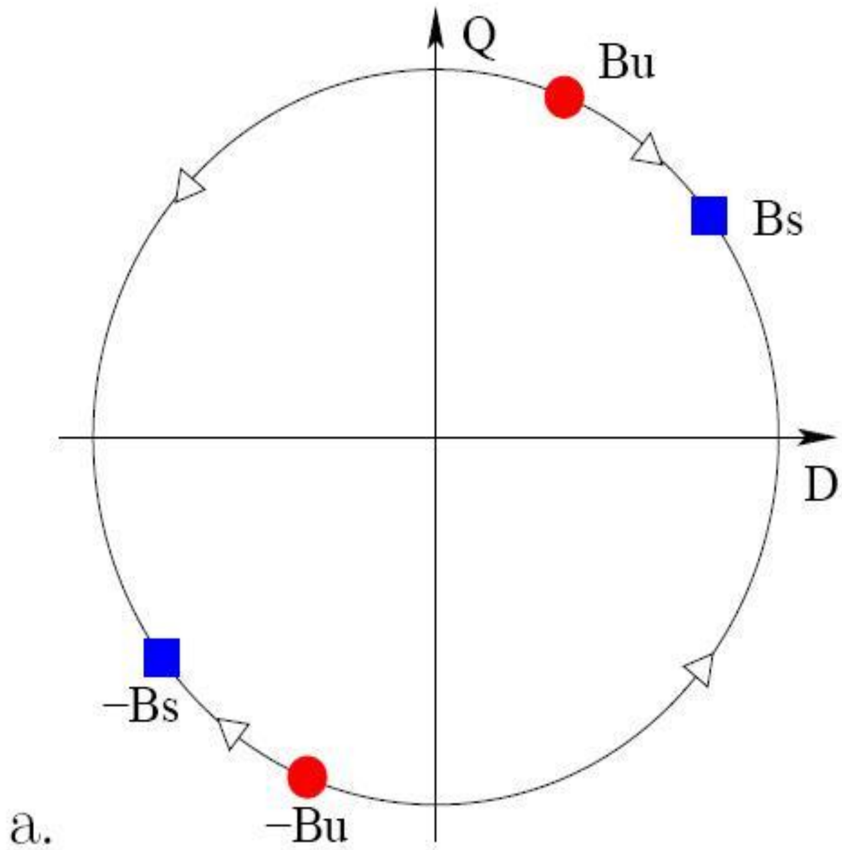
Dipole Quadrupole

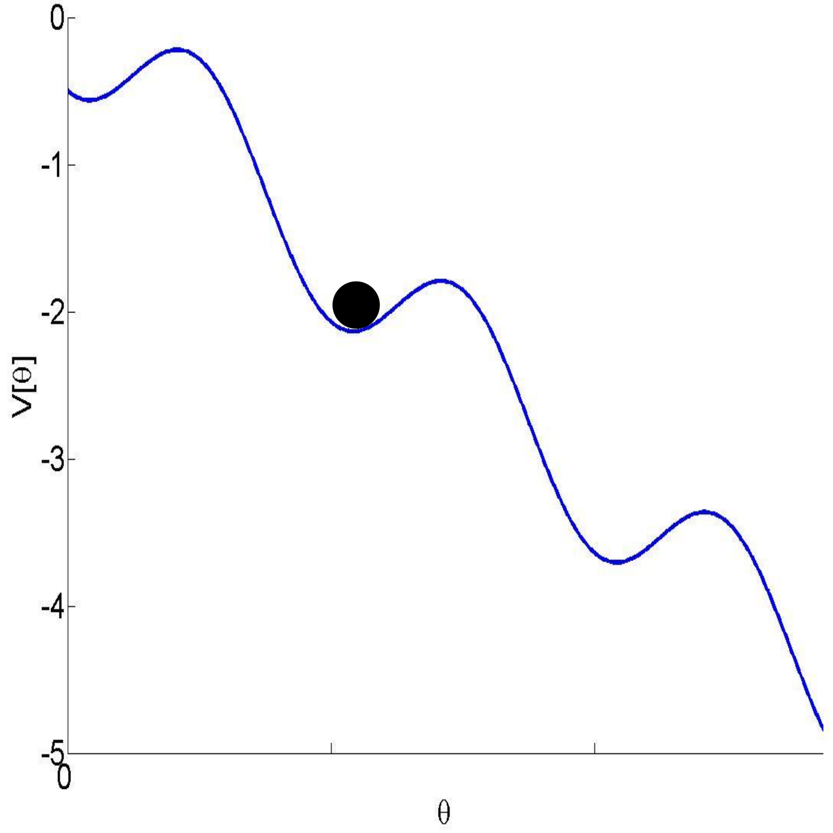
The Earth




VKS

# Effect of turbulent fluctuations: reversals



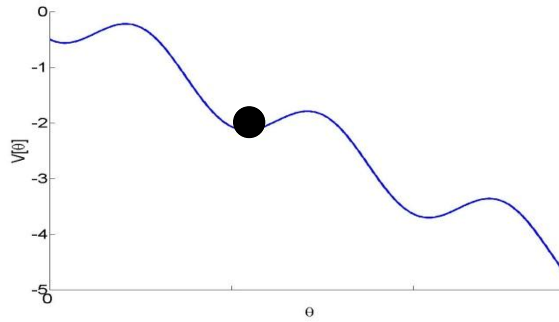
# Predictions

## Mechanism, shape and properties of reversals:

- Two modes are close to a saddle-node bifurcation
- Slow phase followed by a fast phase
- The amplitude of fluctuations required vanishes at the onset of the saddle-node.
- The magnetic field does not vanish, it changes shape.

## Origine and shape of excursions:

- Aborted reversals
- Initial phase similar to reversals, no overshoot at the end




# Statistics of reversals

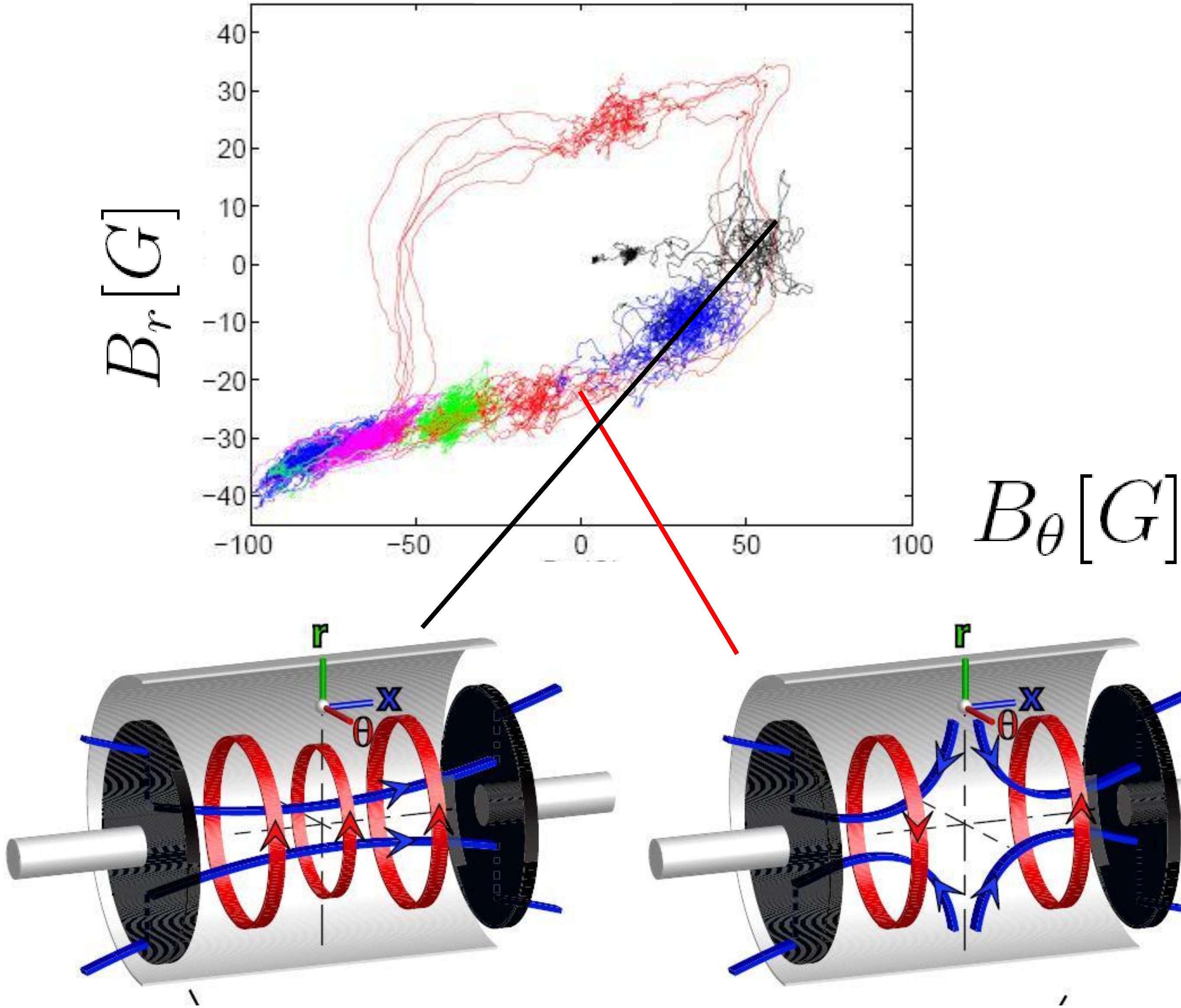
(Excitability close to a saddle-node bifurcation)

$$P[T] = \exp(-T/ < T >) \qquad < T > \propto \exp\left(\Delta V / D\right)$$

Possibility for long phases without reversals

**Comparison with the normal form**

$$\dot{\theta} = \mu_i - \rho \sin(2\theta) + \Delta\zeta(t)$$ **et** $$D = R\cos(\theta + \theta_0)$$

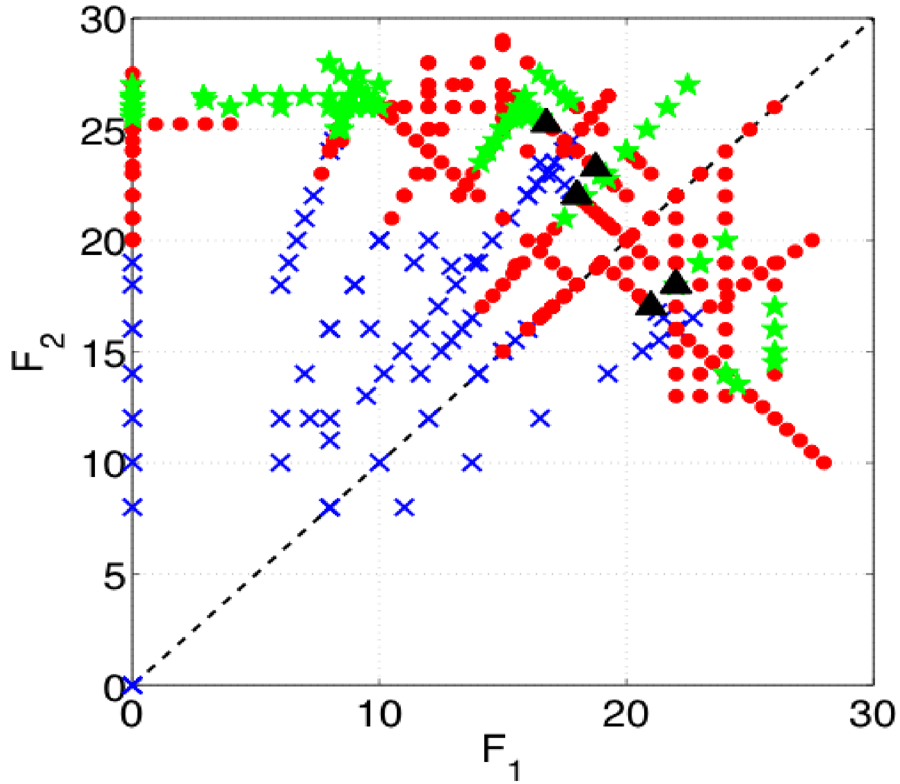



# Dipole and Quadrupole

Ravelet et al. , PRL 101, 074502 (2008)

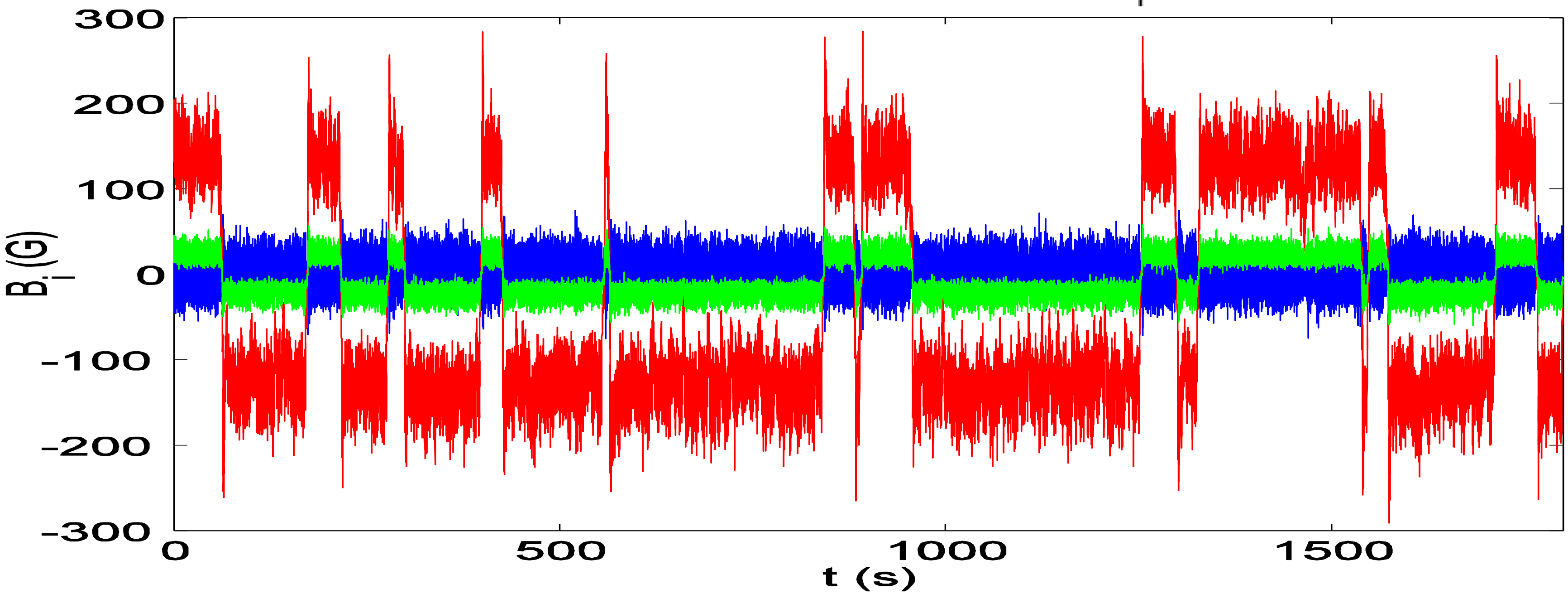
# Parameter space

**(disks rotate at different speeds)**




# A variety of regimes

# (including reversals)



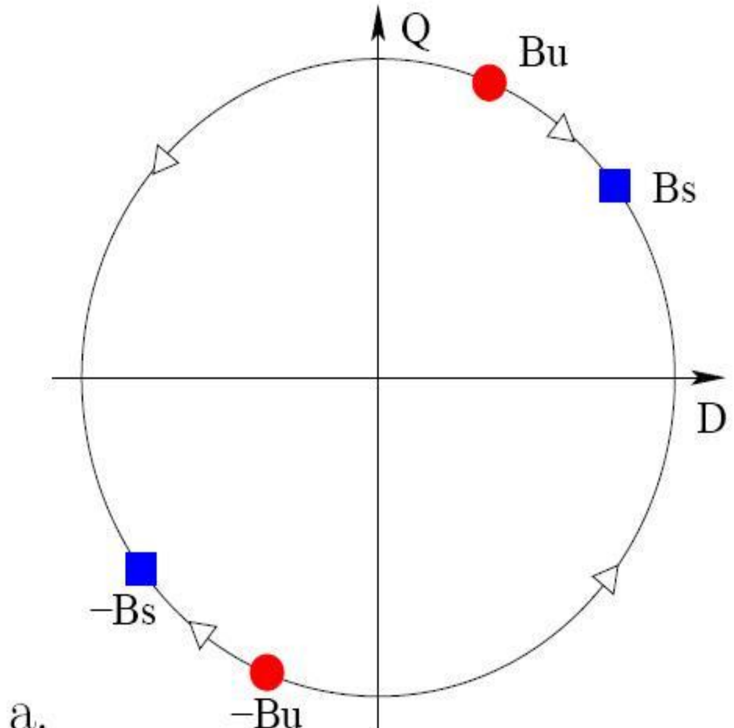
# Mechanism for magnetic field dynamics

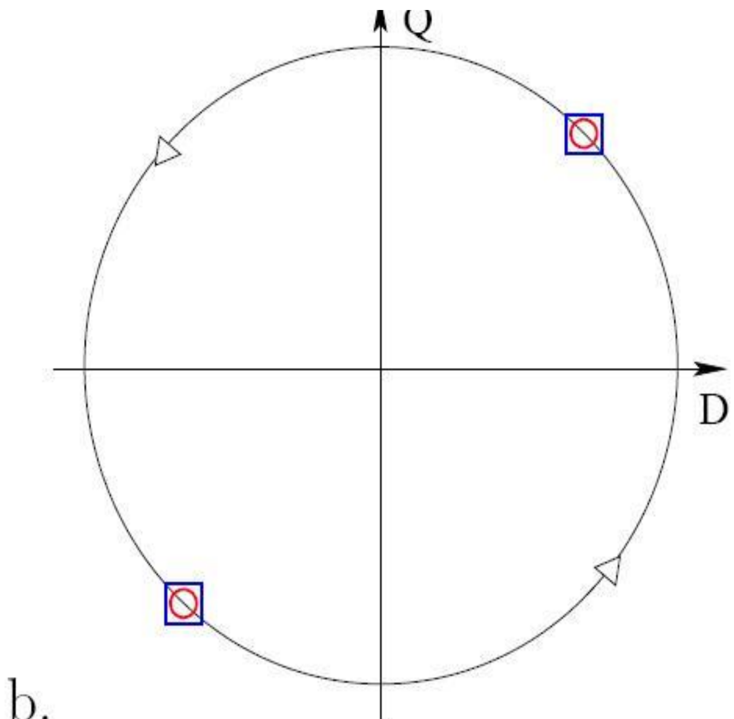
$$\mathbf{B}(r,t) = d(t)\mathbf{D}(r) + q(t)\mathbf{Q}(r)$$

**We set A=d+i q,** $\dot{A} \;=\; \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3$

**Phase equation** $A = r\,\exp\left(i\,\theta\right)$

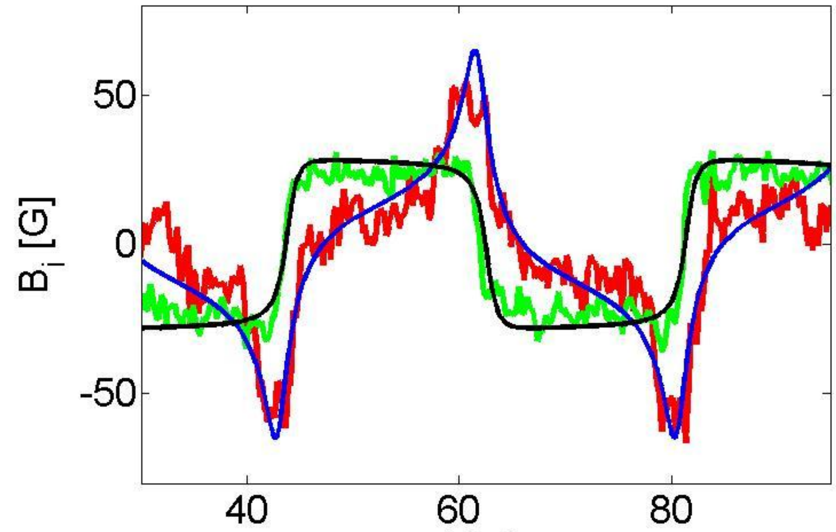
**Simplified expression** $\dot{\theta} = \mu_i - \nu_r \sin\left(2\theta\right)$

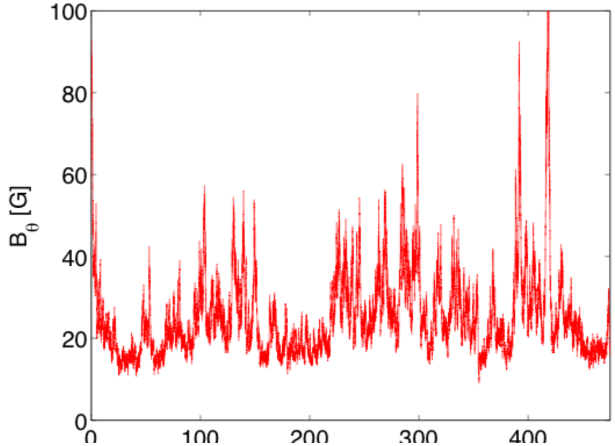


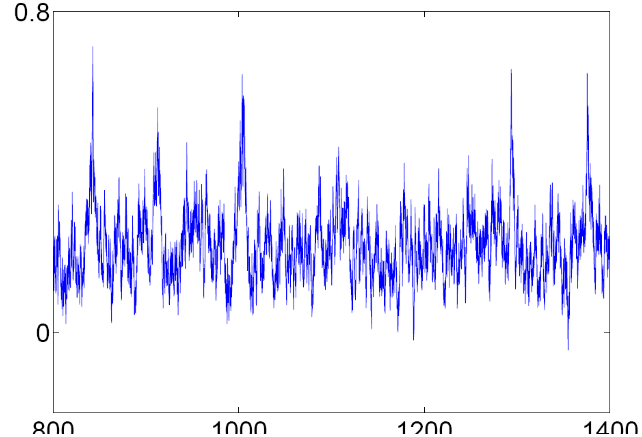
# Comparison

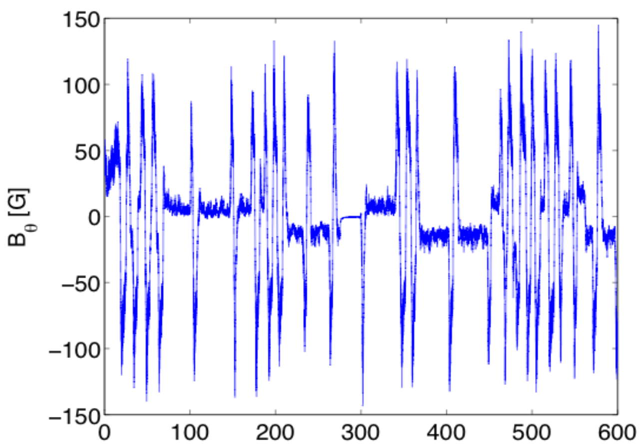
Non-linear oscillations



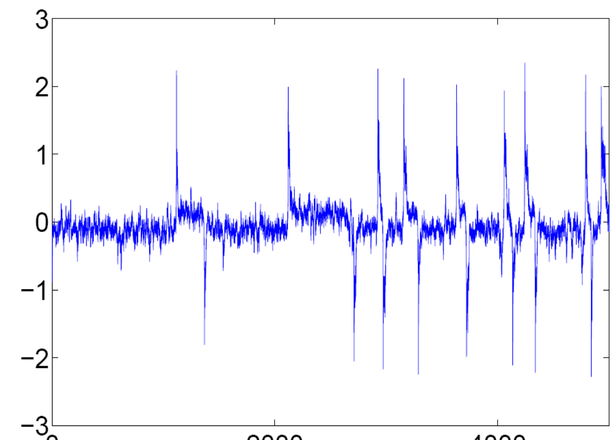




Asymmetric Bursts




Symmetric Bursts



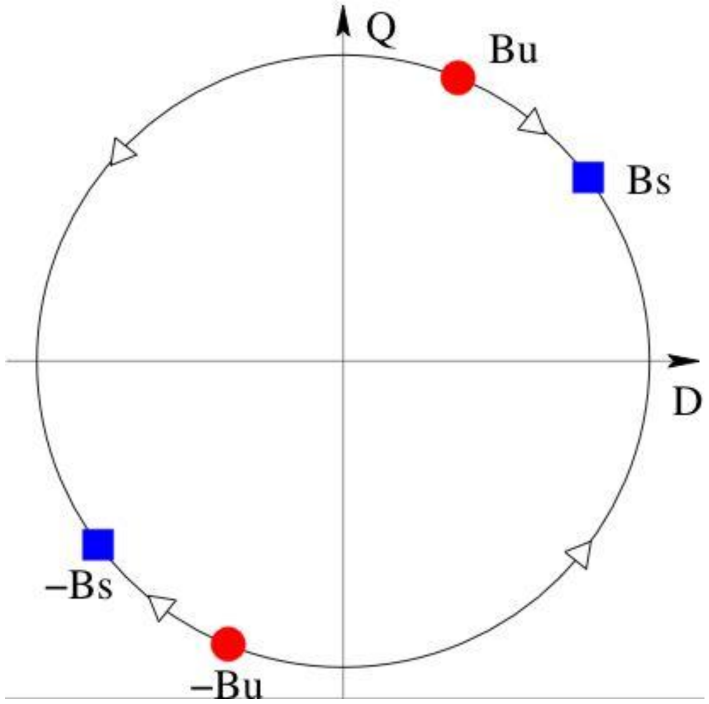
# A similar mechanism for Earth magnetic field

with S. Fauve, E. Dormy (LRA, IPGP) and J.-P. Valet (IPGP)

## Predictions:

Shape, statistics of reversals

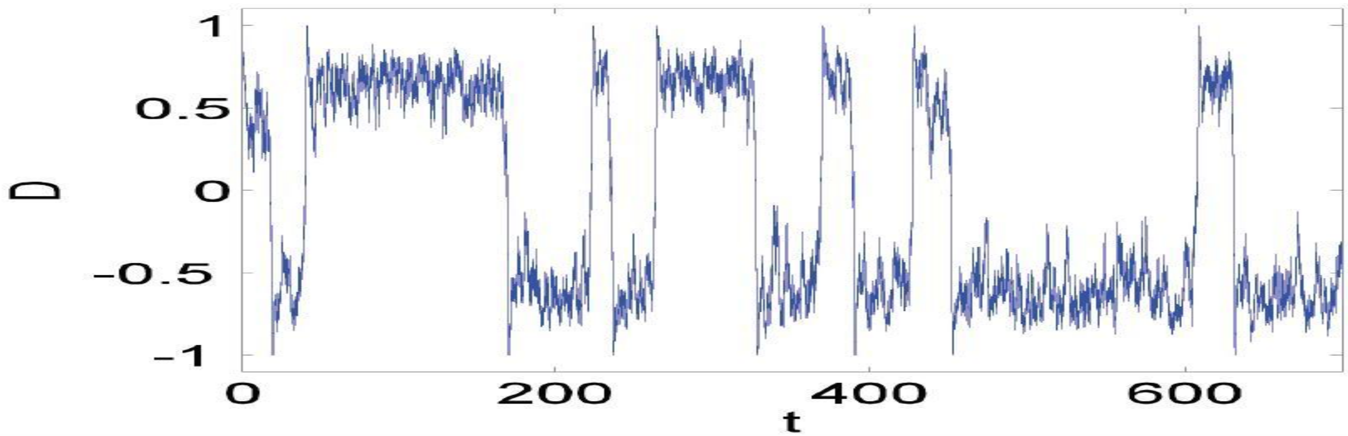
Existence and shape of excursions



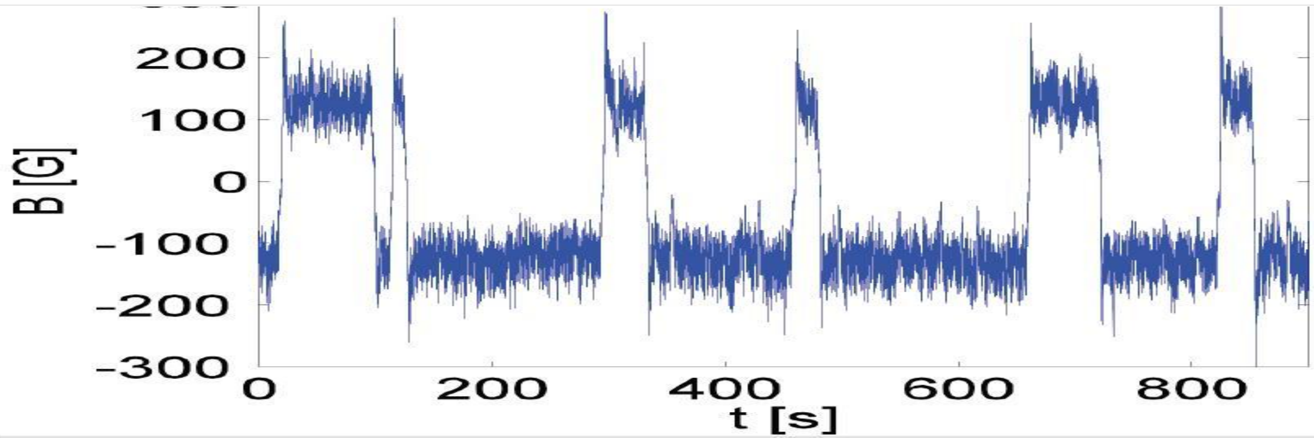

## **Comparison with the normal form**

$$\dot{\theta} = \mu_i - \rho \sin(2\theta) + \Delta\zeta(t) \quad \textbf{and} \quad D = R\cos(\theta + \theta_0)$$

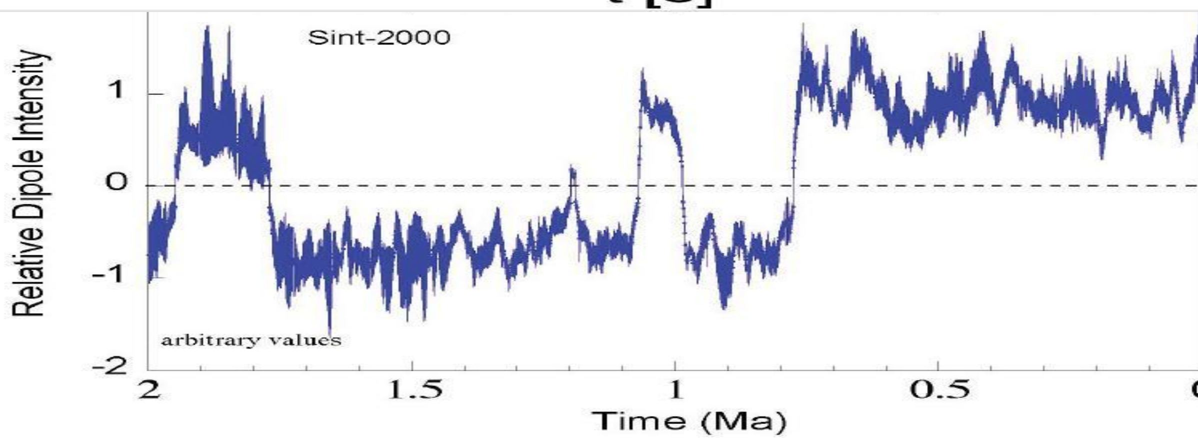
Model




VKS




Earth Dipole

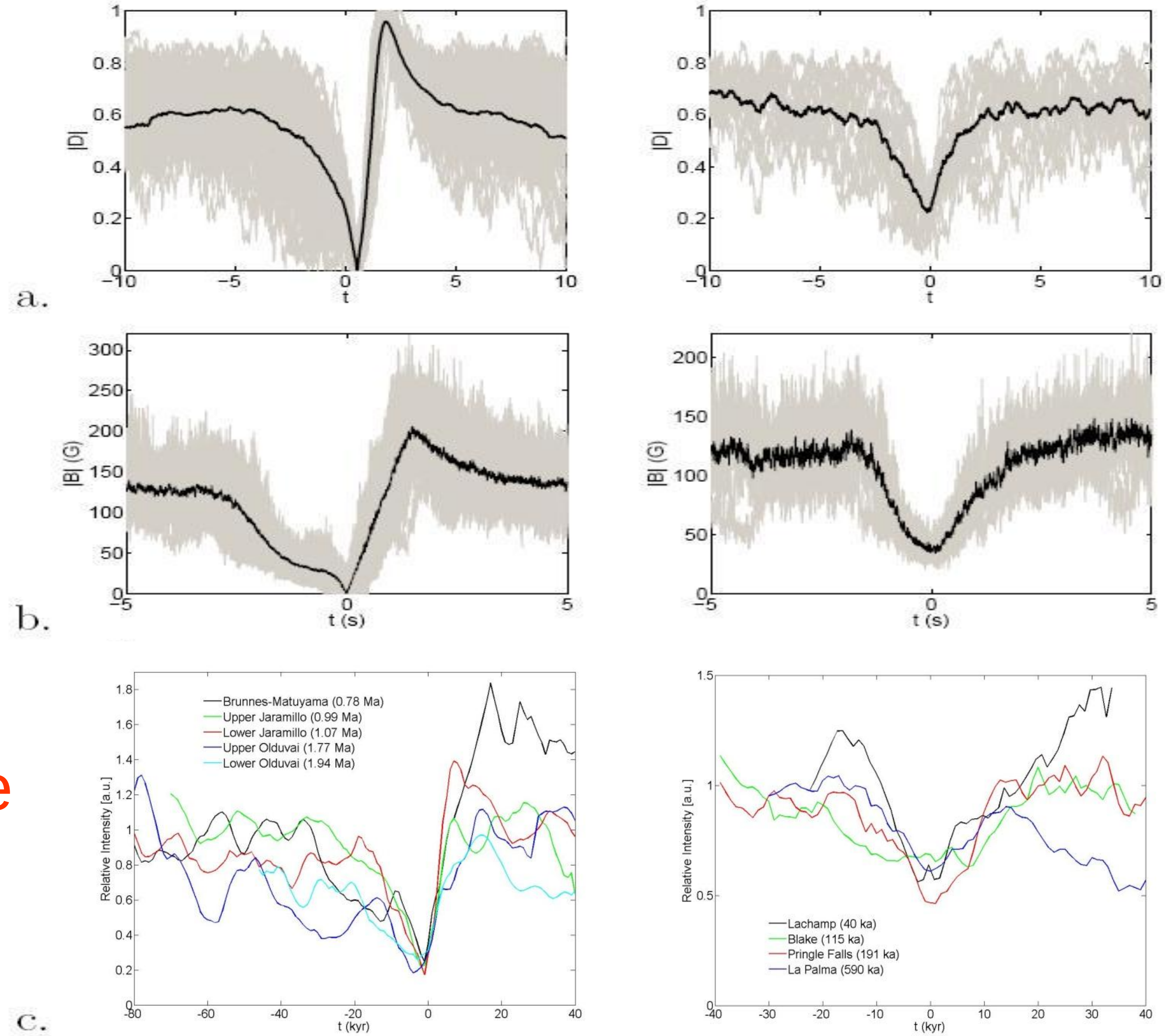


Reversals | Excursions

Model

VKS

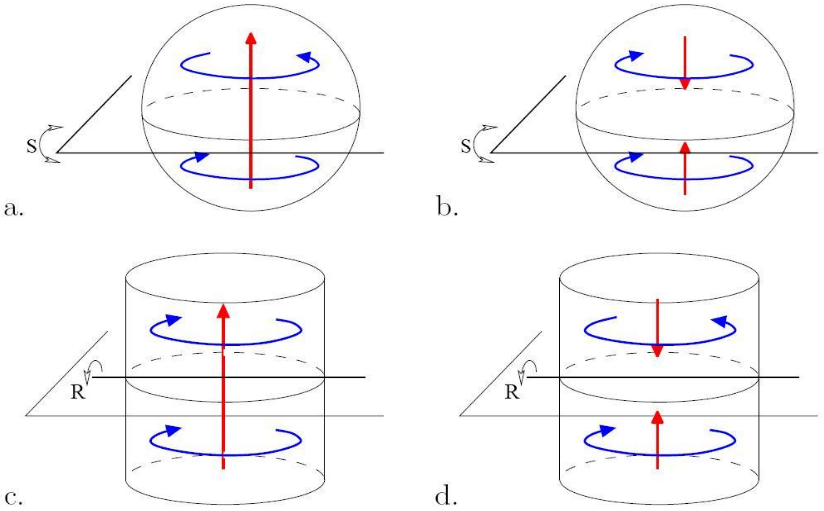
Earth Dipole



# Bifurcation is generic

**For the Earth and VKS,**

**a dipole and a quadrupole**




**Observed in analytical calculations (B. Gallet) and numerical simulations (C. Gissinger)**

## Projects:

- **Caracterisation of the modes**

  Velocity measurements in Gallium (Berhanu, Gallet, Mordant)

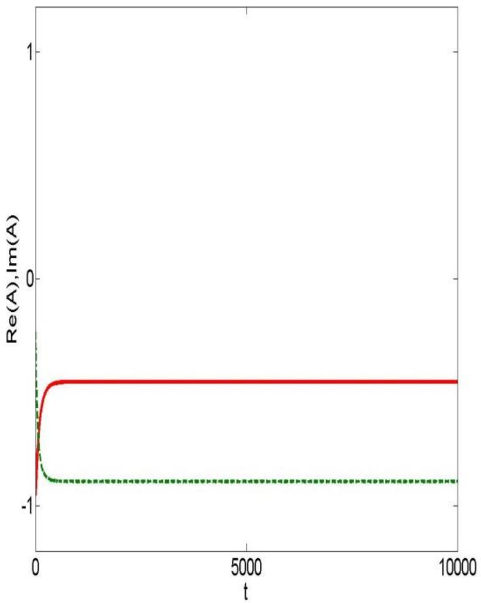
- **Dynamo without iron disks**

  An optimized flow for alpha-omega effect

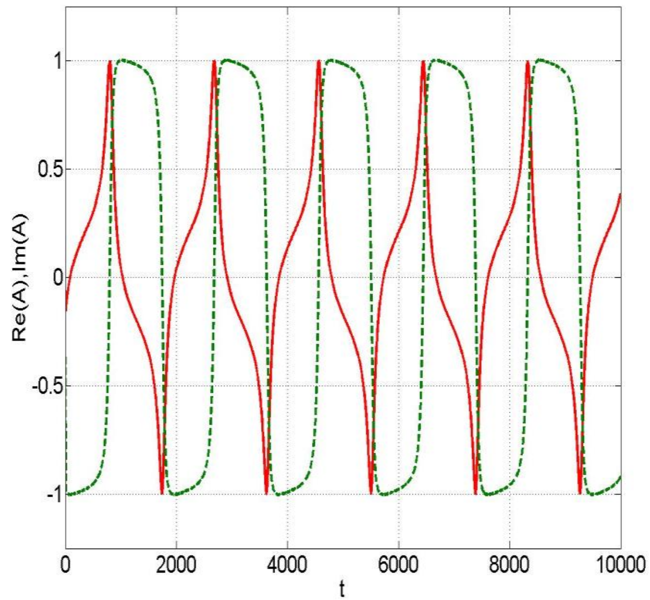
- **Reversals in other systems**

If F1=F2: coefficients are real
coupling cannot drive the saddle-node bifurcation

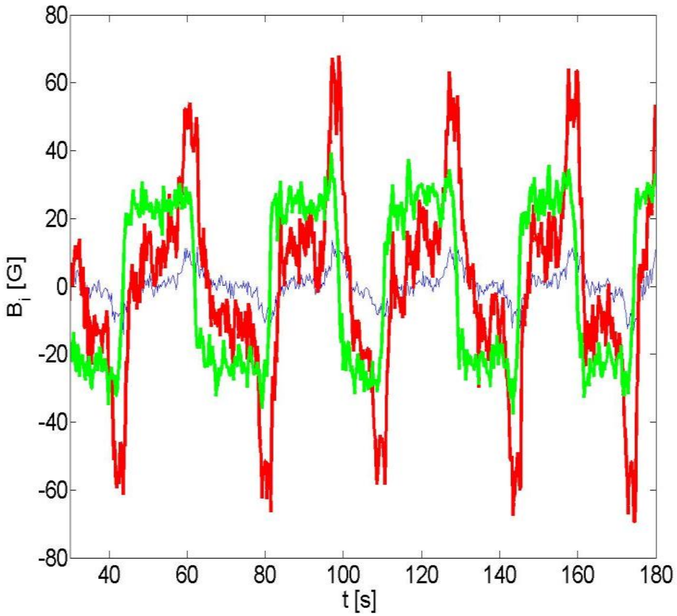
Examples of time-series obtained (coefficients are prescribed functions of f $\alpha$ F1-F2):




f=0.5




f=1.05




VKS