Selection, large deviations and metastability

1. Dynamics with selection

- **A cell performs complex dynamics: DNA codes for the production of proteins, which themselves modify how the reading is done. A bit like a program and its RAM content.**
- **DNA contains about the same amount of information as the TeXShop program for Mac**
- **This dynamics admits more than one attractor: same DNA yields liver and eye cells...**
- **The dynamical state is inherited.**
- **On top of this process, there is the selection associated to the death and reproduction of individual cells**

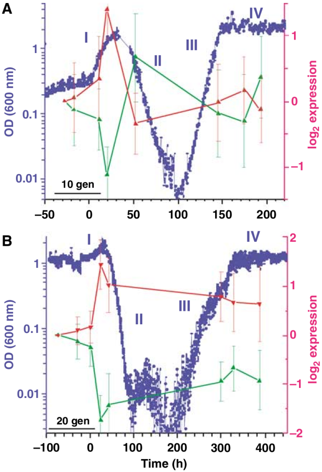
Stern, Dror, Stolovicki, Brenner, and Braun

An arbitrary and dramatic rewiring of the genome of a yeast cell:

the presence of glucose causes repression of histidine biosynthesis, an essential process

Cells are brutally challenged in the presence of glucose, nothing in evolution prepared them for that!

Stern, Dror, Stolovicki, Brenner, and Braun



Stern, Dror, Stolovicki, Brenner, and Braun

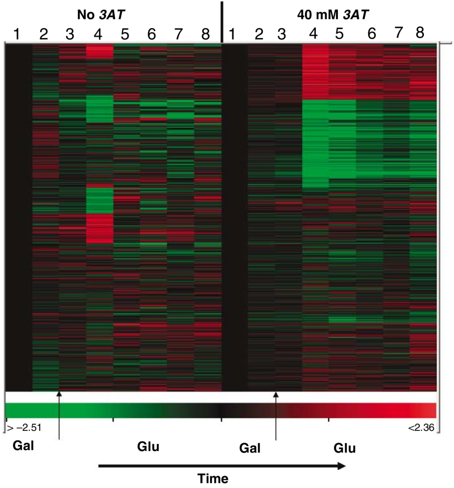


Figure 2 The genome-wide transcription pattern. The raw transcription levels at eight time points for the two experiments, (left) no *3AT*, (right) 40 mM *3AT*, in a color code: red—induced, green—repressed. There are a total of 4148 genes that passed all filters (see Materials and methods). The medium switch from galactose to glucose is marked and the numbers above the columns are the measurement points as shown in Figure 1. Note the differences between the patterns of expression for the two experiments (rows correspond to the same gene in both experiments).

- **the system finds a transcriptional state with many changes**
- **two realizations of the experiment yield vastly different solutions**
- **the same dynamical system seems to have chosen a different attractor** which is then inherited over many generations

If this interpretation is confirmed, we are facing a dynamics in a complex landscape

with the added element of selection

but note that fitness does not drive the dynamics, it acts on its results

the landscape is not the 'fitness landscape'

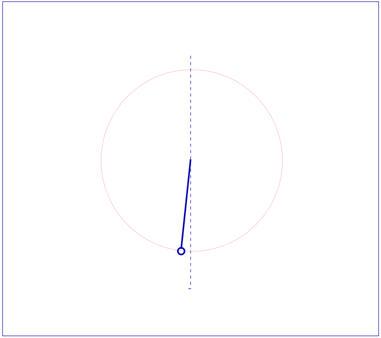
2. The relation between

a) Large Deviations,

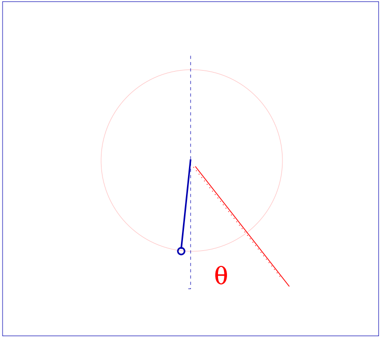
b) Metastability

c) Dynamics with selection and phase transitions

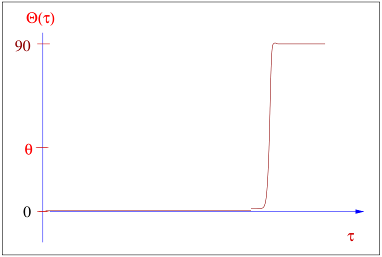
a pendulum immersed in a low-temperature bath



a pendulum immersed in a low-temperature bath



Imposing the average angle, the trajectory shares its time between saddles 0^o and 180^o



phase-separation is a first order transition!

$$\int D[\theta]P(\text{trajectory})\ \ \delta\left[\int_0^t\ \theta(t')\ dt' - t\theta_o\right]$$

$$= \int d\lambda\ \ \underbrace{\int D[\theta]P(\text{trajectory})\ \ e^{\lambda\int_0^t\ \theta(t')\ dt'}}_{canonical}\ \ e^{-\lambda t\theta_o}$$

canonical version, with λ conjugated to θ

$$Z(\lambda) = \int D[\theta]P(\text{trajectory})\ \ e^{\lambda\int_0^t\ \theta(t')\ dt'}$$

- λ **is fixed to give the appropriate** θ **(Laplace transform variable)**

- **a system of walkers with cloning rate** $\lambda\theta(t)$

$$\frac{dP}{dt} = -\left\{\frac{d}{d\theta}\left(T\frac{d}{d\theta} + \sin(\theta)\right)\right\} P - \lambda\theta\ P$$

yields the 'canonical' version of the large-deviation function

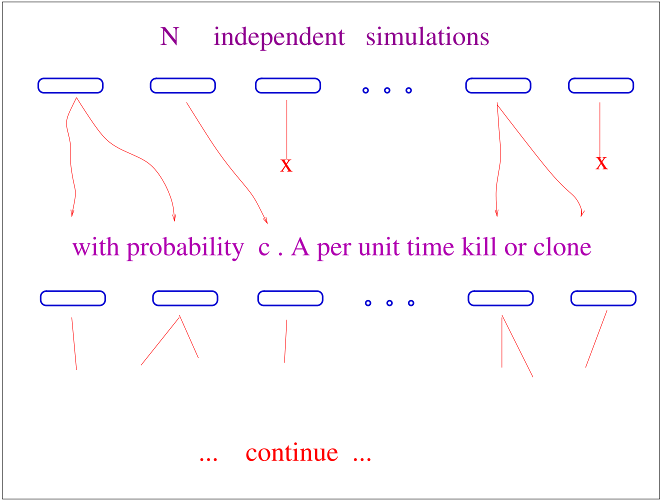
- **the relation is useful for efficient simulations**

- **but also to understand the large deviation function**

We wish to simulate an event with an unusually large value of A

without having to wait for this to happen spontaneously

but without forcing the situation artificially



a way to count trajectories weighted with e^{cA}

- A collection of metastable states

- **each with its own emigration rate**

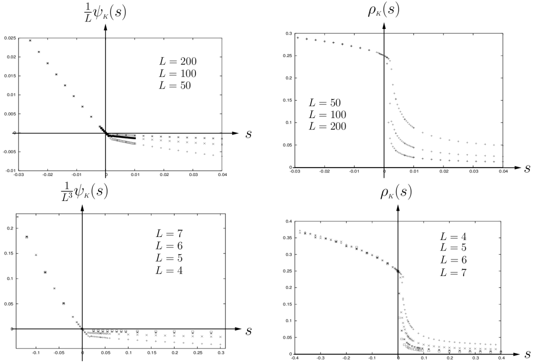
- **and its cloning/death rates dependent upon the observable**

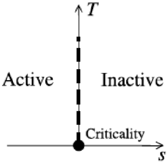
We understand the relation between metastability and large deviations

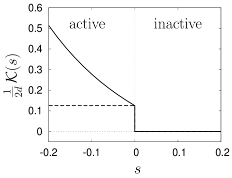
Dynamical phase transitions

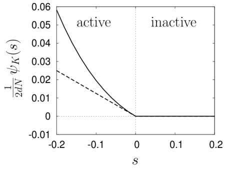
large deviations of the activity

JP Garrahan, RL Jack, V Lecomte, E Pitard, K van Duijvendijk, and Frederic van Wijland







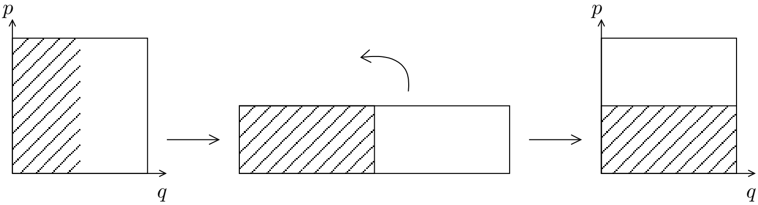


may be obtained with selection proportional to the activity

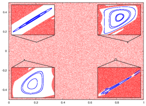
3. Large deviations and order

What do extremal trajectories look like?

The baker's map

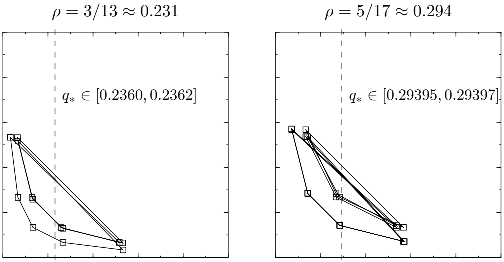


... is as chaotic as you can be.



And yet, orbits minimising a function, e.g.

$$\mathcal{A} \equiv \int dt\ (q(t) - q_*)^2$$



are periodic or quasiperiodic but unstable!

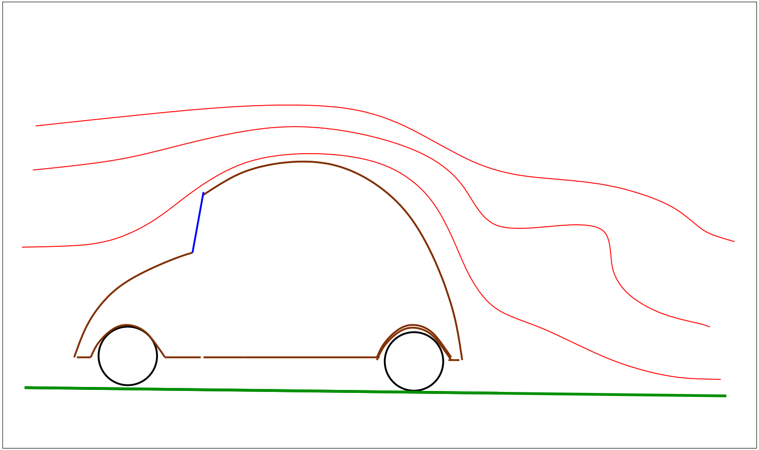
Hunt and Ott — Khan-Dang Nguyen Thu Lam, JK , D Levine

In a case like this one:

T. Duriez, J..L. Aider, E. Masson, J.E. Wesfreid ; Qualitative investigation of the main flow features over a TGV ; Proceedings of the Euromech Colloquium 509, Vehicle Aerodynamics, Berlin, Allemagne, **2009**, p. 52-57 http://opus.kobv.de/tuberlin/volltexte/2009/2249/

$$\bar{f}_\tau = \frac{1}{\tau} \int_0^\tau f(t)\, dt$$

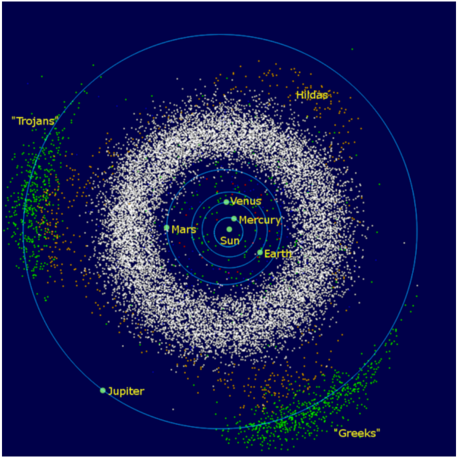
if this metaphor is good, we should see order



during exceptional times

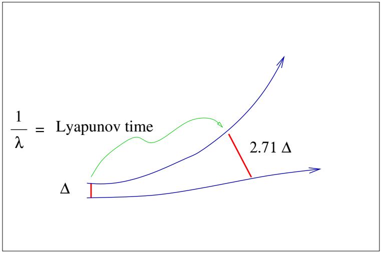
4. Miscellanea

advertisement for J. Tailleur's talk



because planets disturb one another, the dynamics is chaotic

chaotic?



difference between trajectories multiplies by $e = 2.71...$ every $\sim 5M$ years Laskar

$\lambda \equiv 1/5MYrs$ **is called the *Lyapunov exponent***

$\lambda > 0 \quad \rightarrow$ **chaos**

Consider this:

- If you start a planetary random system in your computer, it often runs into trouble.
- If you observe a planetary system, many conditions within the observational error imply recent formation or immediate destruction
- Are there places between large planets where earth-like planets may have relative stability?

You need to know rare trajectories

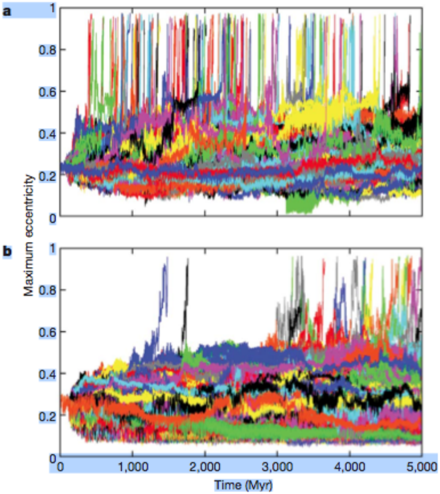
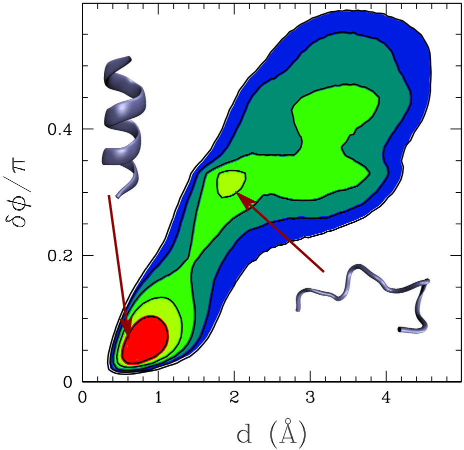


Figure 1 | Mercury's eccentricity over 5 Gyr. Evolution of the maximum

Laskar et al

The problem of transitions

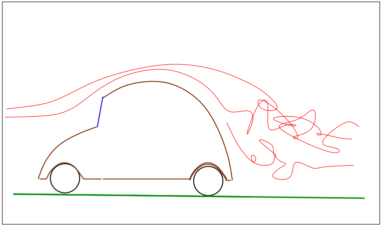


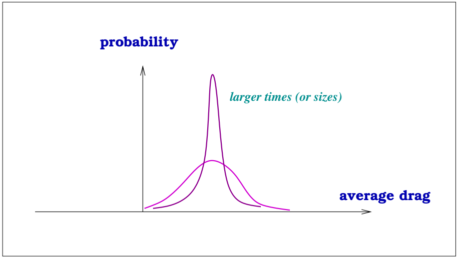
peptide helix-coil transition

The problem of transitions may be shown to be

a problem of large deviation functions of the (largest) Lyapunov exponent!

Drag, traffic jams, etc:





$$\bar{f}_\tau = \frac{1}{\tau} \int_0^\tau f(t)\, dt$$

Intermittency in fully developed turbulence:

Longitudinal-structure functions

$$S_p(R) = \langle |\mathbf{v}(\mathbf{x}+\mathbf{R}) - \mathbf{v}(\mathbf{x})|^p \rangle = \langle e^{\overbrace{p \ln |\mathbf{v}(\mathbf{x}+\mathbf{R}) - \mathbf{v}(\mathbf{x})|}} \rangle$$

who is responsible for the large moments?