

Bistability and transitions induced by  
topography in a laboratory model of a  
geostrophic jet

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# Generic features of geophysical turbulence

- Shallow-water flows dominated by rotation - predominantly two-dimensional
- Organization into large-scale structures like jets, vortices
- Abrupt qualitative changes in these large-scale structures
- Atmospheric blocking (Weeks *et al.* 1997)

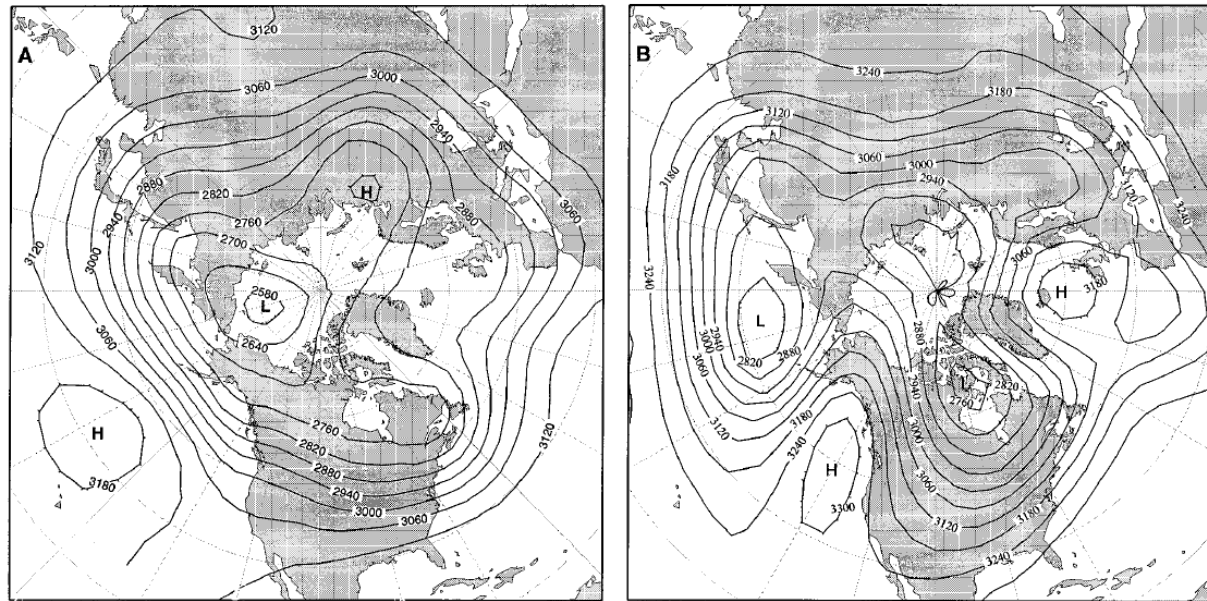
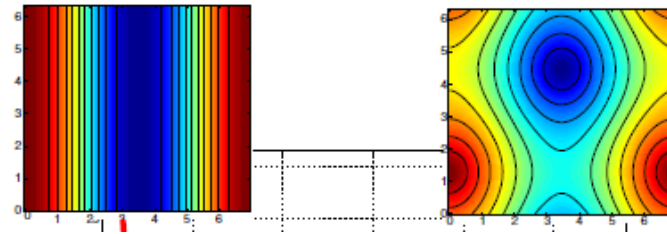


Fig. 1. Atmospheric pictures of (A) zonal and (B) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric

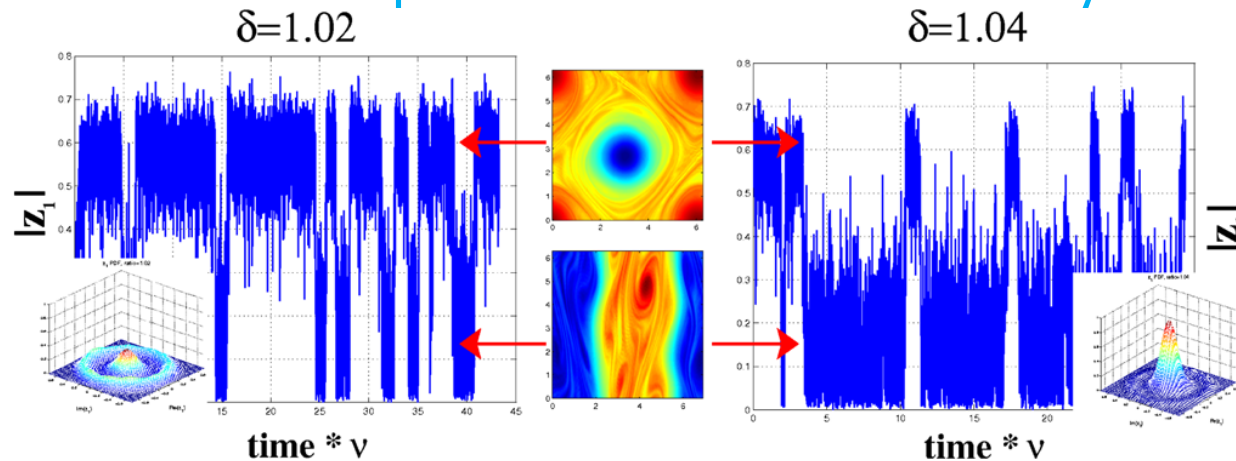
Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

# 2D NS Equations (Bouchet & Simmonet 2009)

- 2D Euler equations on a doubly periodic domain
  - Equilibrium statistical mechanics predicts a 2<sup>nd</sup> order phase transition between unidirectional and dipole flows



- Adding stochastic forcing and dissipation takes the system away from equilibrium – 1<sup>st</sup> order phase transition - bistability



- 2D NS equations are structurally similar to more realistic models (quasi-geostrophic) of geophysical flows

# Bistability of the Kuroshio Current

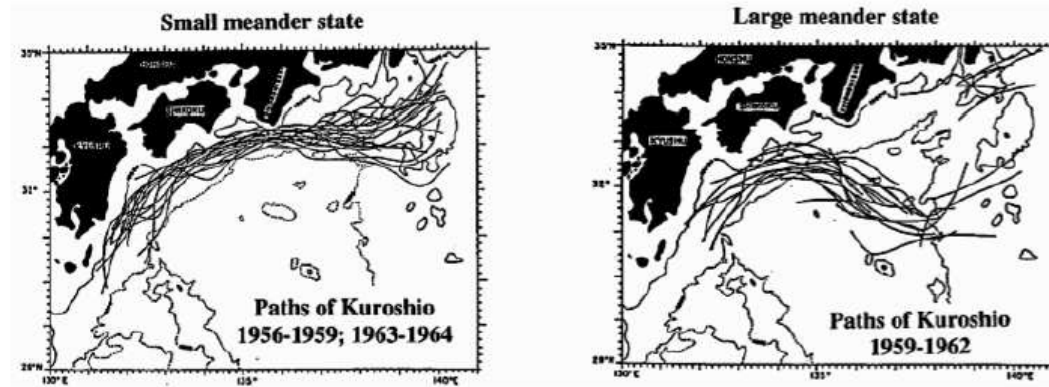


Figure 3: Bistability of the paths of the Kuroshio during the 1956-1962 period : paths of the Kuroshio in (left) its small meander state and (right) its large meander state. The 1000-m (solid) and 4000-m (dotted) contours are also shown. (figure from Schmeits and Dijkstra [47], adapted from Taft 1972.)

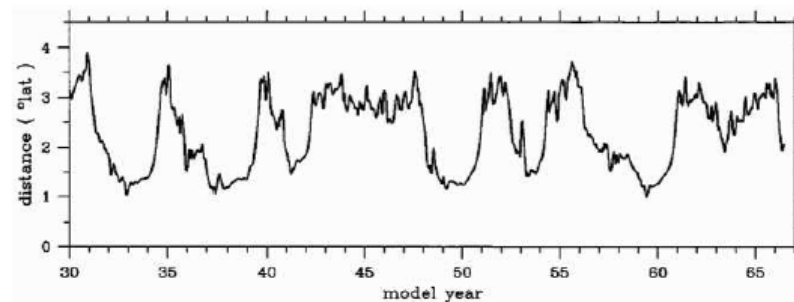
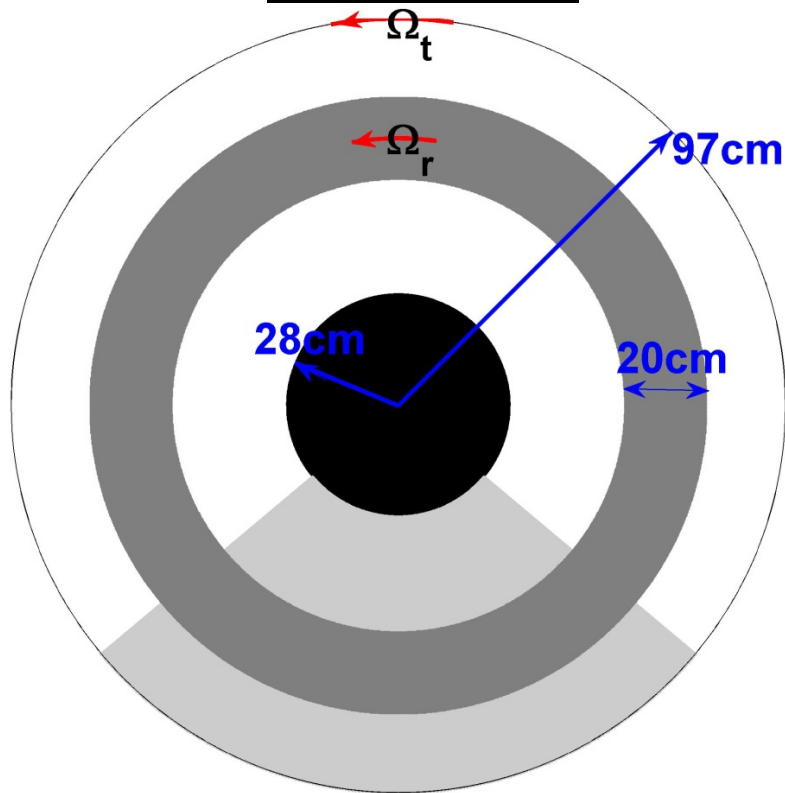


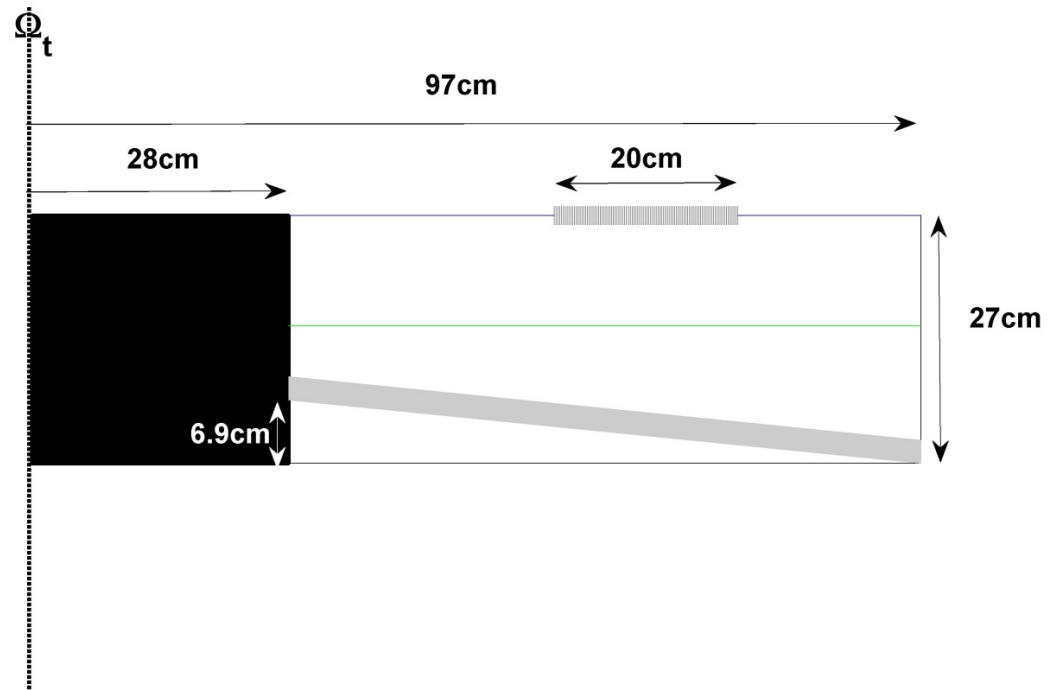
Figure 4: Bistability of the paths of the Kuroshio, from Qiu and Miao [41] : timeseries of the distance of the Kuroshio jet axes from the coast, averaged over the part of the coast between 132°-140°E, from a numerical simulation using a two layer primitive equation model.

# Experimental Set-up - Schematic

Top View



Side View



Typical Values:  $\Omega_t \approx 0.4 - 0.6 \text{ rad/s}$

$$\Omega_r \approx 0.78 \text{ rad/s}$$

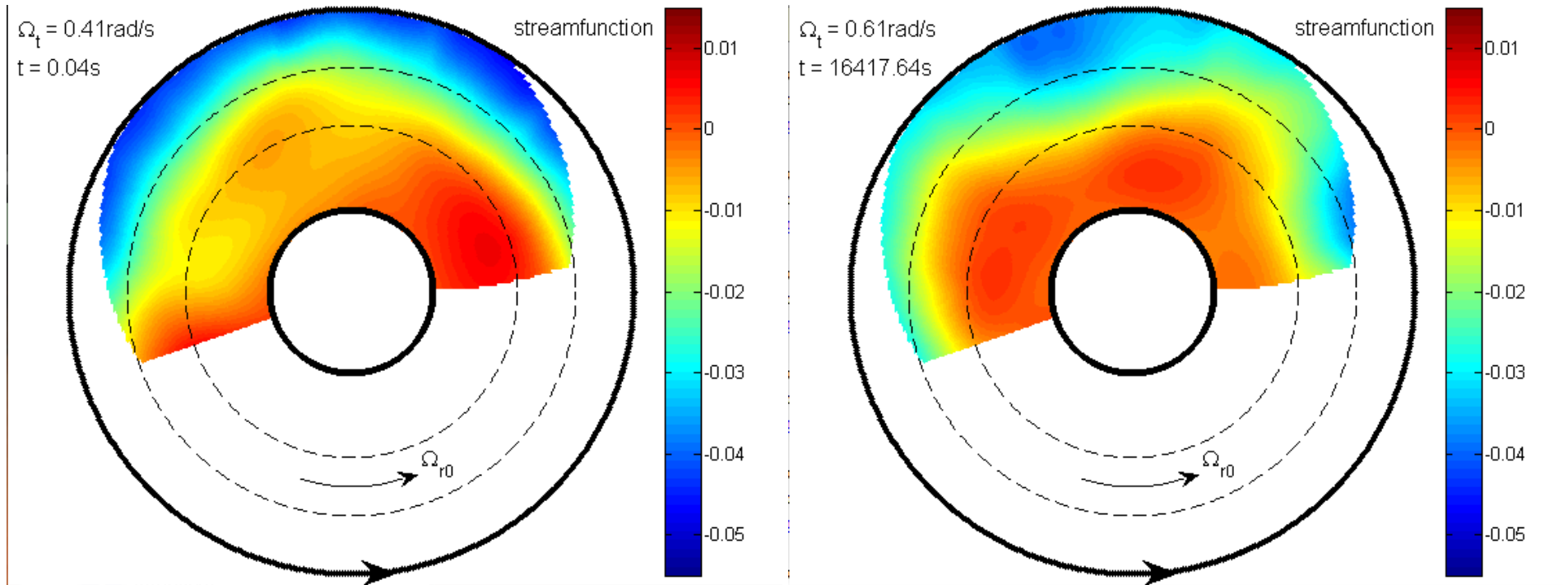
$$\beta = \frac{2\Omega_t s}{H} \approx 0.37 \text{ rad/s/m}$$

$$Ro = \frac{U}{2\Omega_t L} \approx 2.6$$

$$Ek = (4\pi/H)^2 (\nu/\Omega_t) \approx 0.0043$$

$$L_R = \frac{(gH)^{1/2}}{2\Omega_t} \approx 1.63 \text{ m}$$

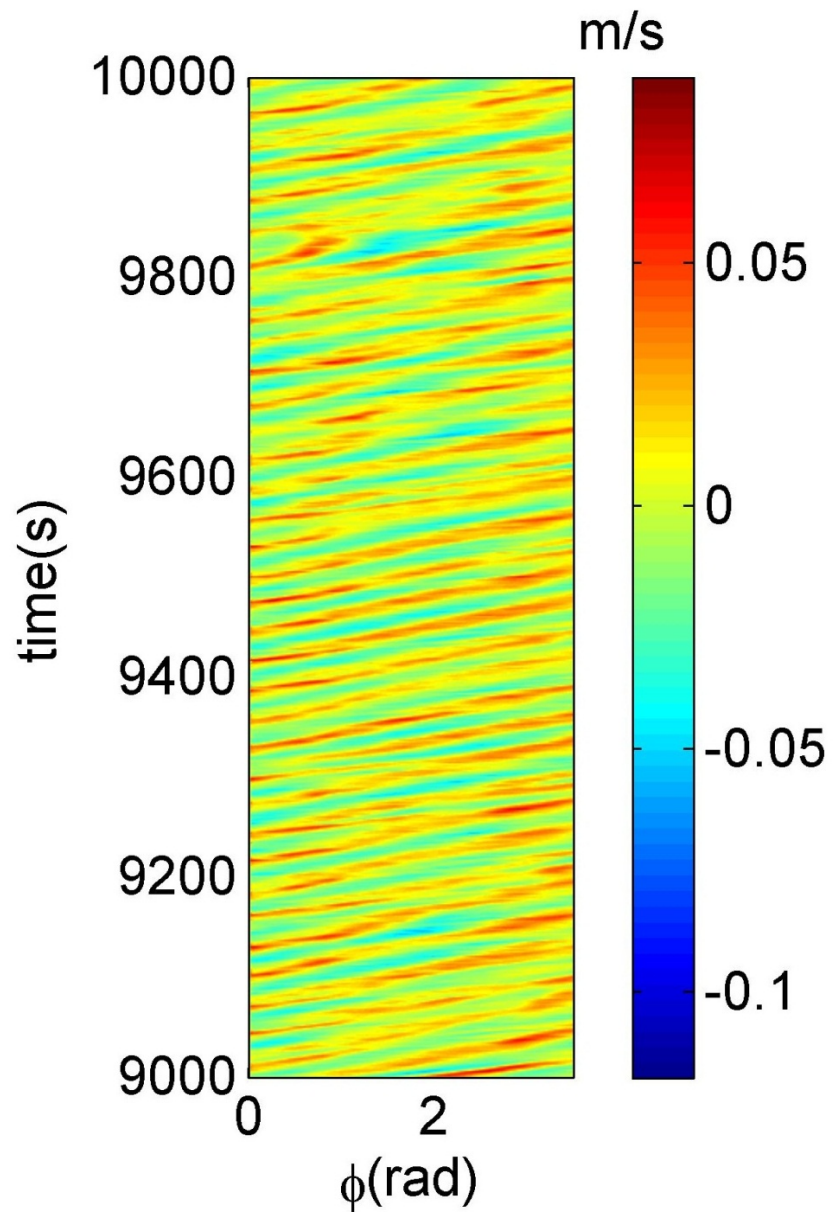
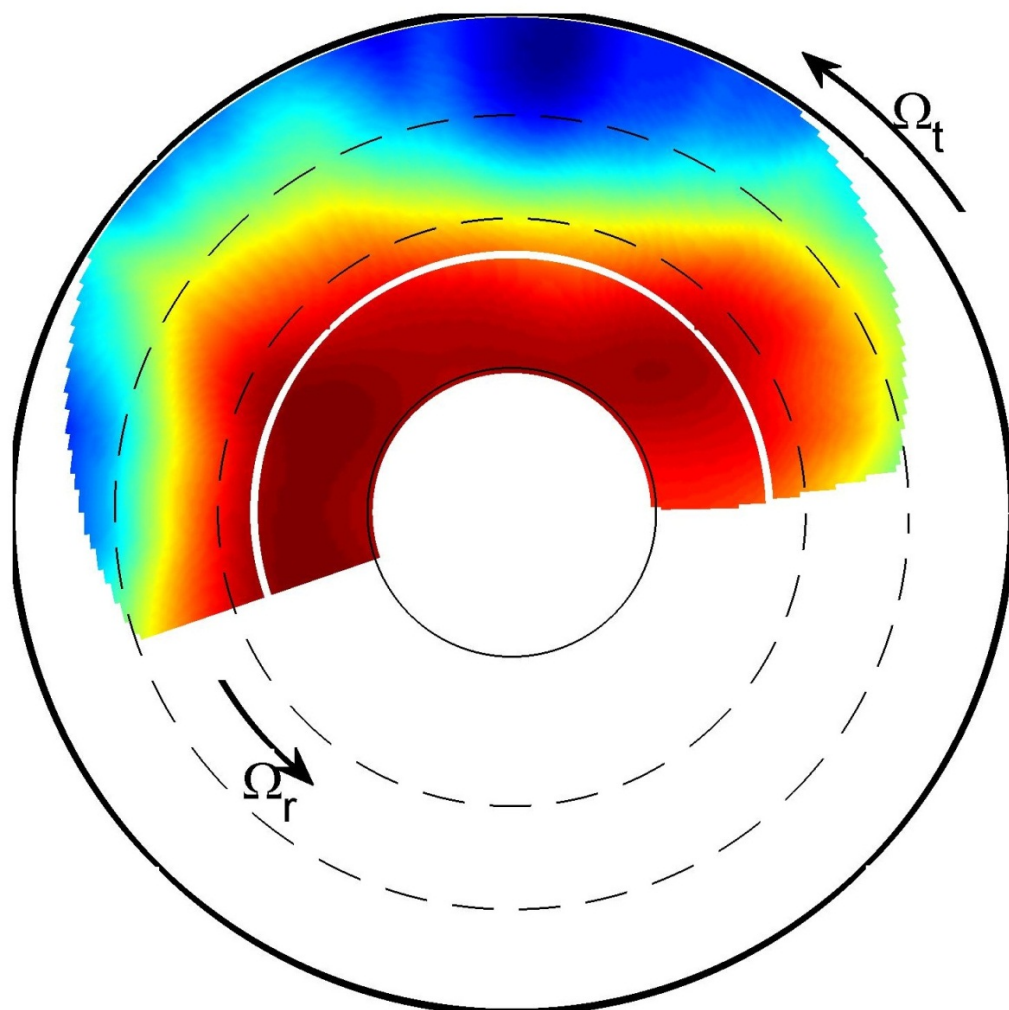
# NO TOPOGRAPHY



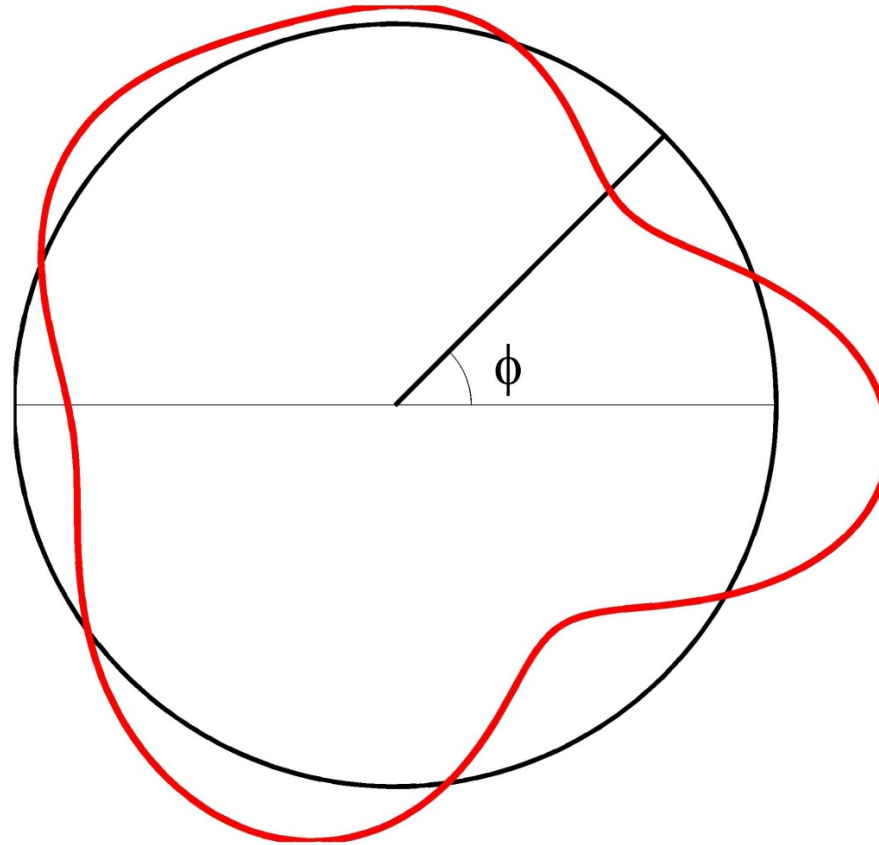
- Axi-symmetry broken by a barotropic instability of the jet
- Propagating waves evident in both the scenarios
- What does a sweep over the entire range of  $\Omega_t$  give ?

# NO TOPOGRAPHY - $\Omega_t \uparrow$

streamfunction



# ANALYSIS METHOD



$$f(\phi) = A_1 e^{i\phi_1} e^{i\phi} + A_2 e^{i\phi_2} e^{2i\phi} + A_3 e^{i\phi_3} e^{3i\phi} + \dots$$

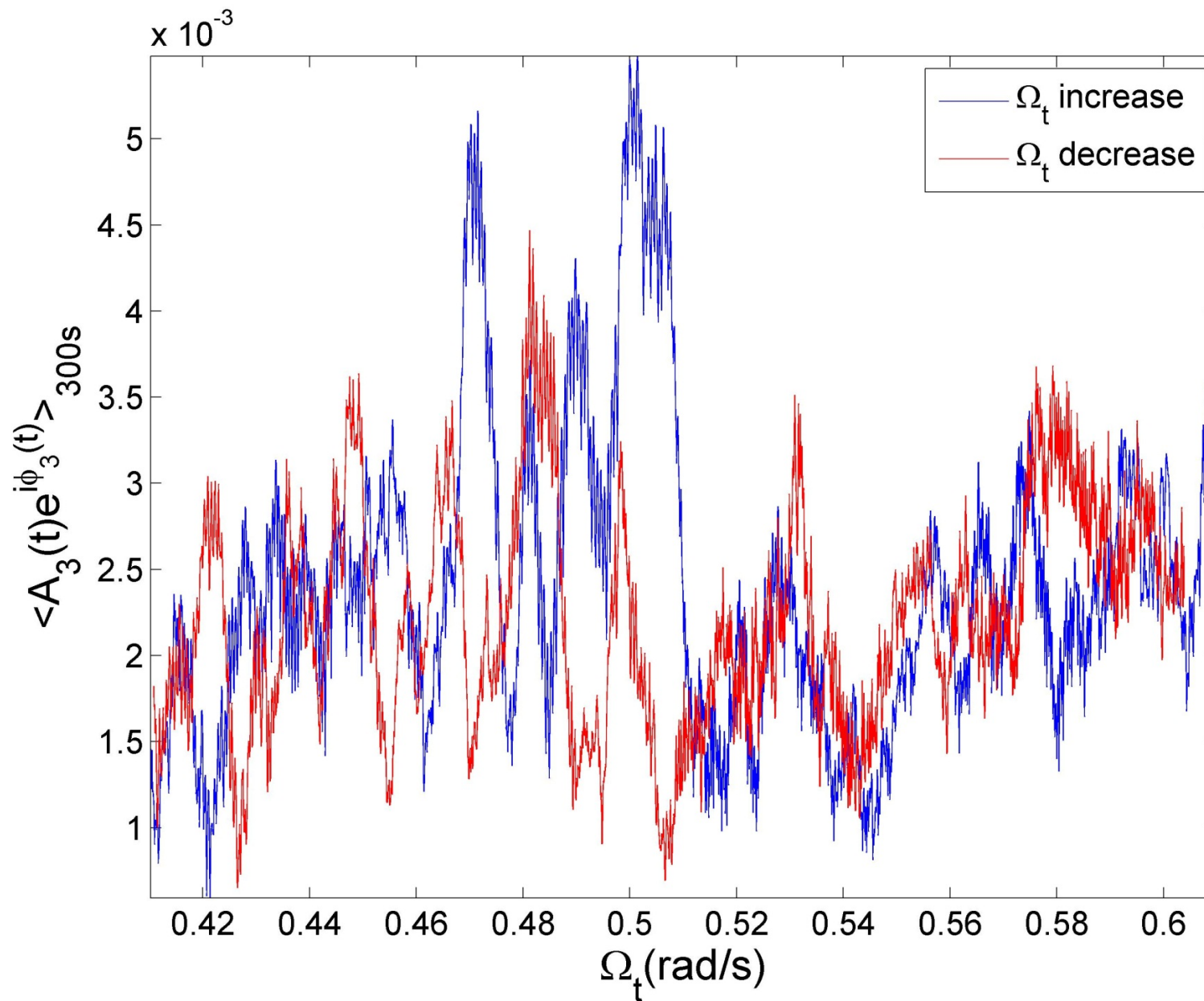
$$f(\phi, t) = A_1(t) e^{i\phi_1(t)} e^{i\phi} + A_2(t) e^{i\phi_2(t)} e^{2i\phi} + A_3(t) e^{i\phi_3(t)} e^{3i\phi} + \dots$$

purely propagating mode-3 wave:  $\langle A_3(t) e^{i\phi_3(t)} \rangle = 0$

standing mode-3 wave:  $\langle A_3(t) e^{i\phi_3(t)} \rangle = c_0$



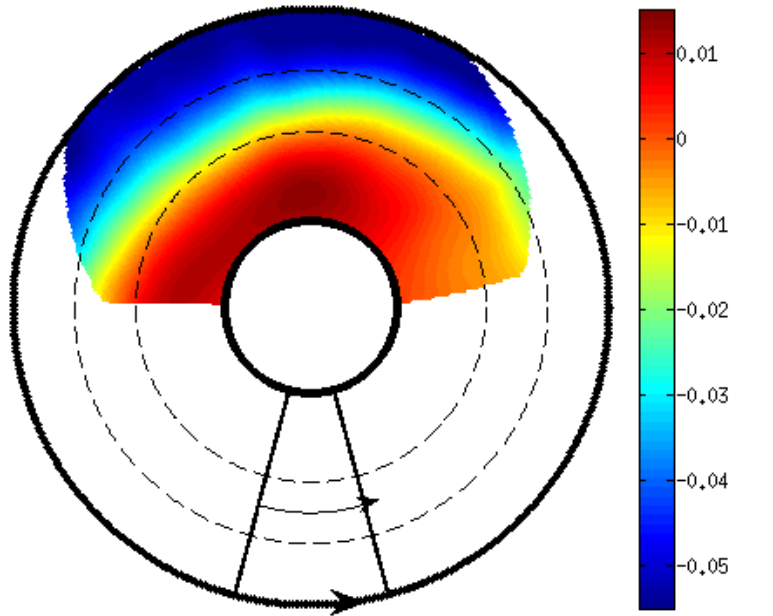
# NO TOPOGRAPHY – SWEEP OVER $\Omega_t$



# WITH TOPOGRAPHY – SWEEP OVER $\Omega_t$

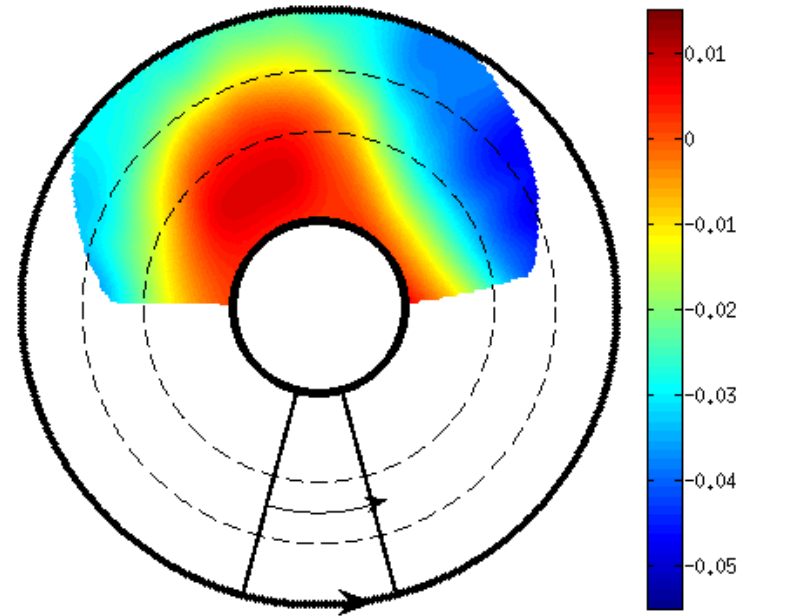
$\Omega_t$  increase

$\omega_t = 0.44\text{rad/s}$ , time = 0.04s



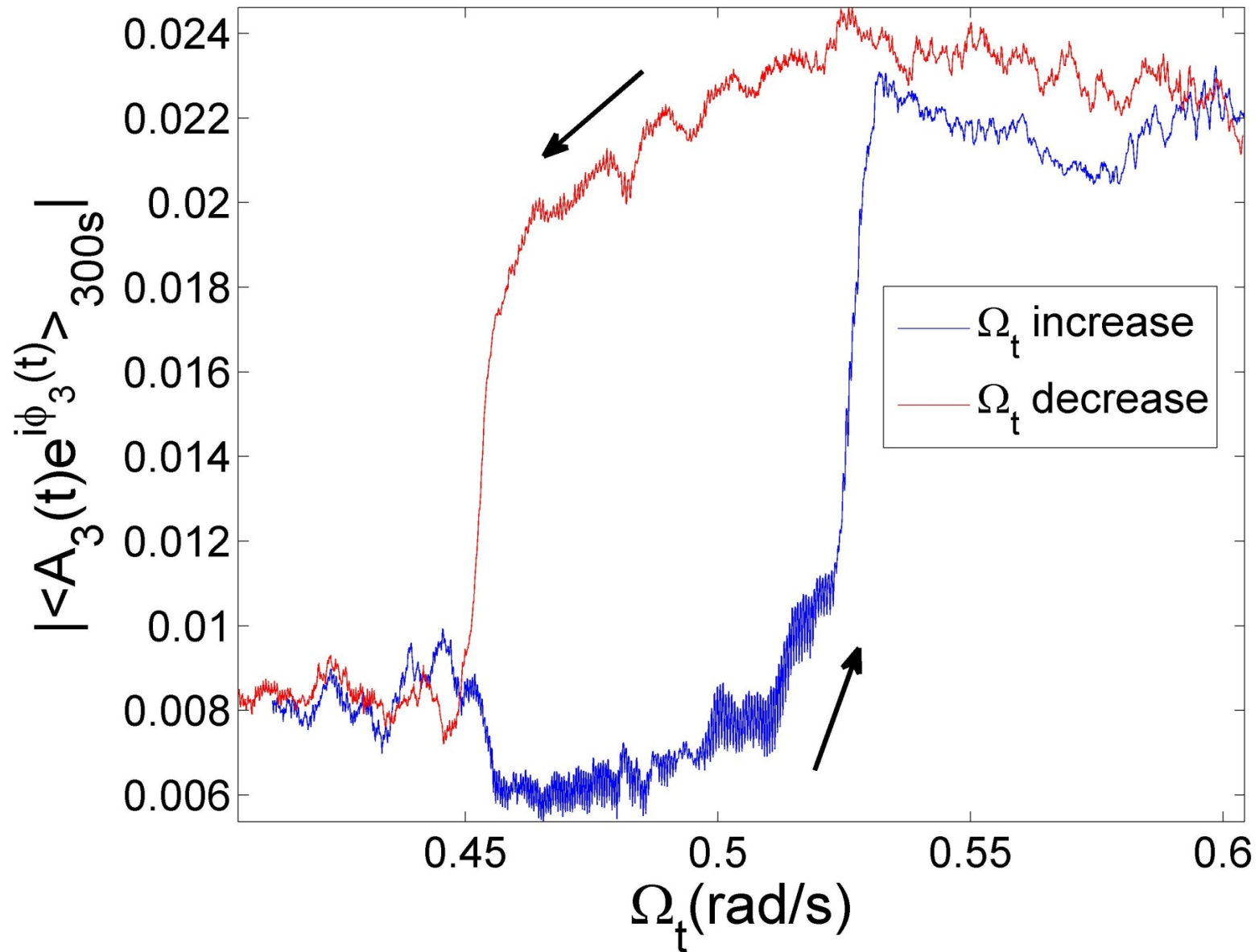
$\Omega_t$  decrease

$\omega_t = 0.59\text{rad/s}$ , time = 0.04s

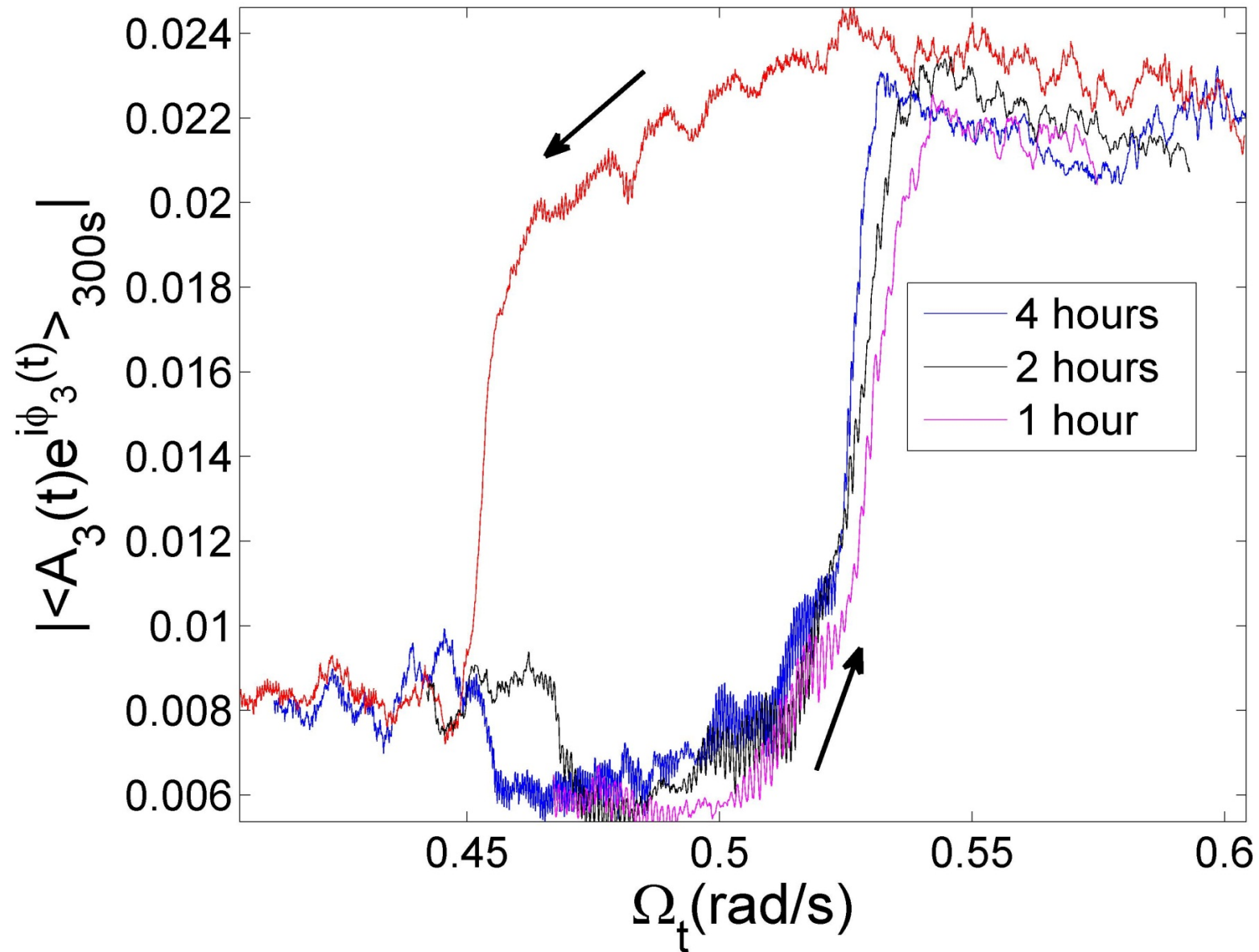


- Transitions occur at different points in the two experiments

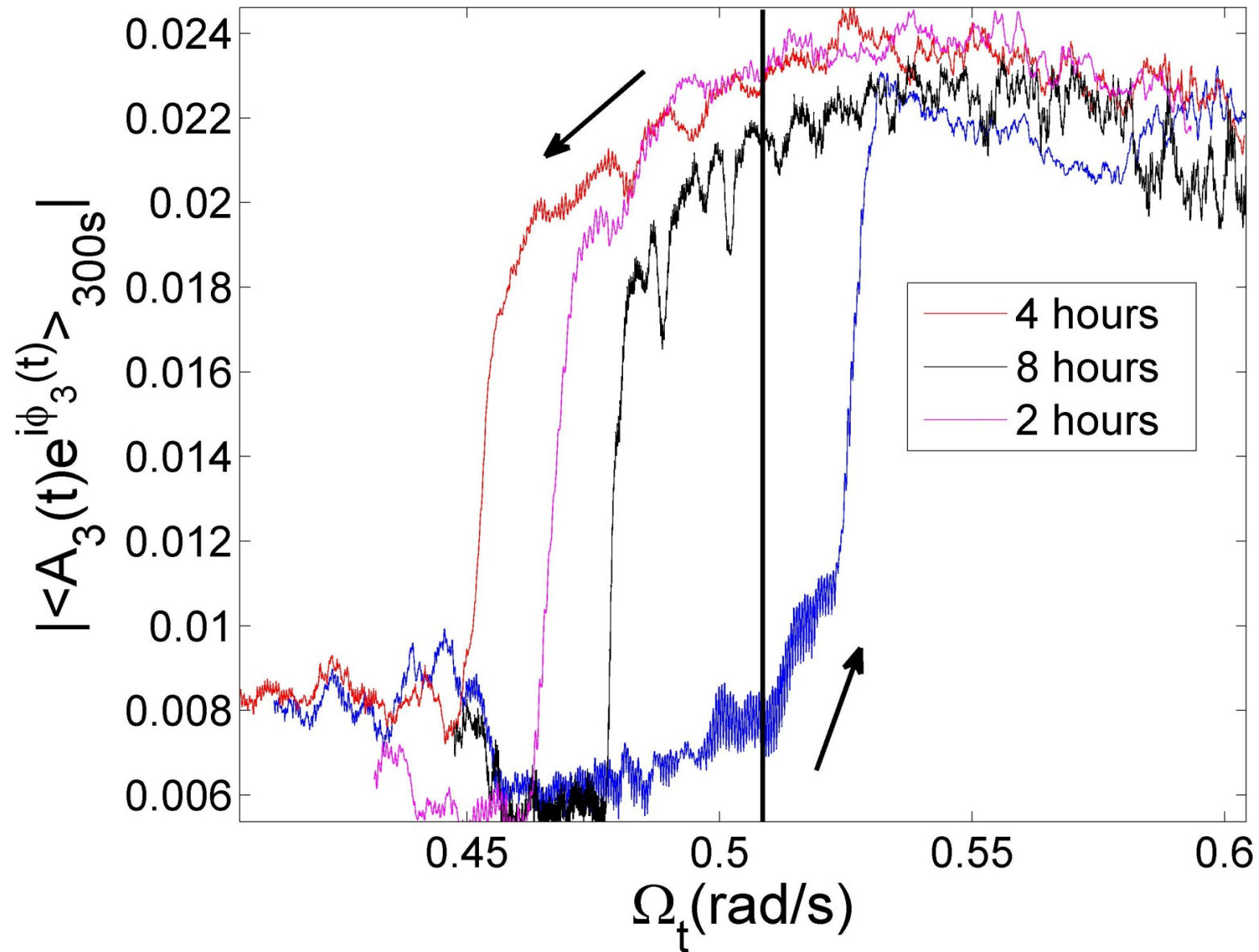
# WITH TOPOGRAPHY – SWEEP OVER $\Omega_t$



# WITH TOPOGRAPHY – SWEEP OVER $\Omega_t$

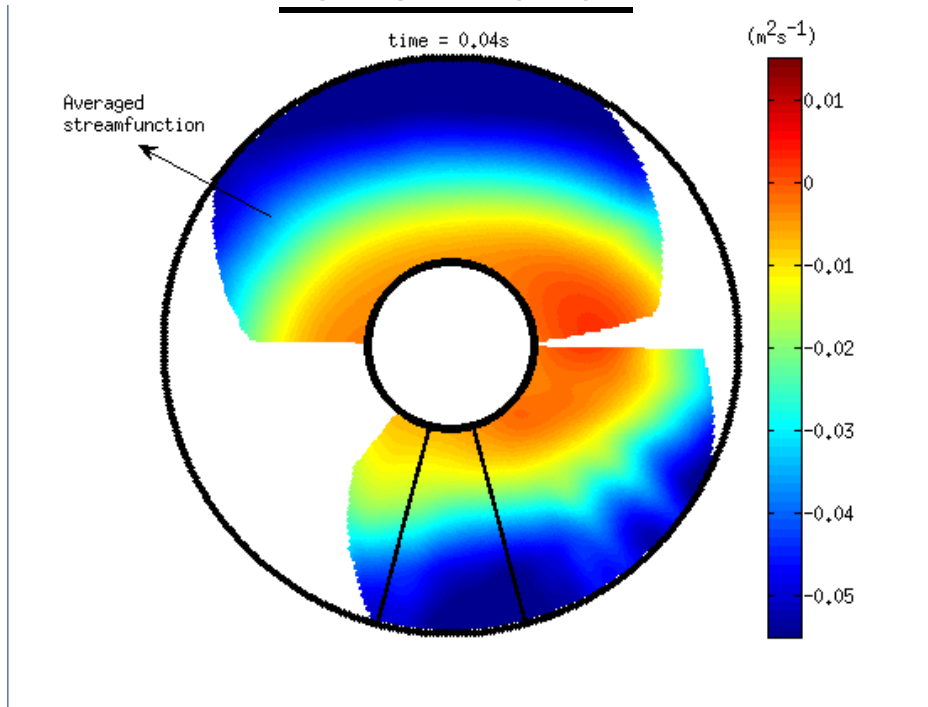


# WITH TOPOGRAPHY – SWEEP OVER $\Omega_t$

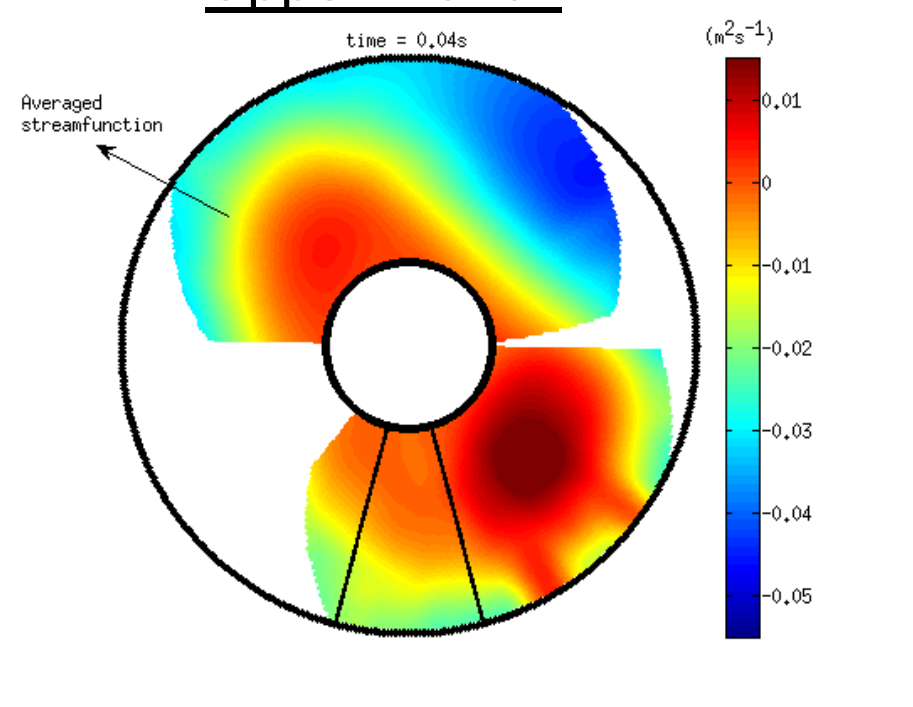


$$\underline{\Omega}_t = 0.51 \text{ rad/s}$$

### Lower Branch

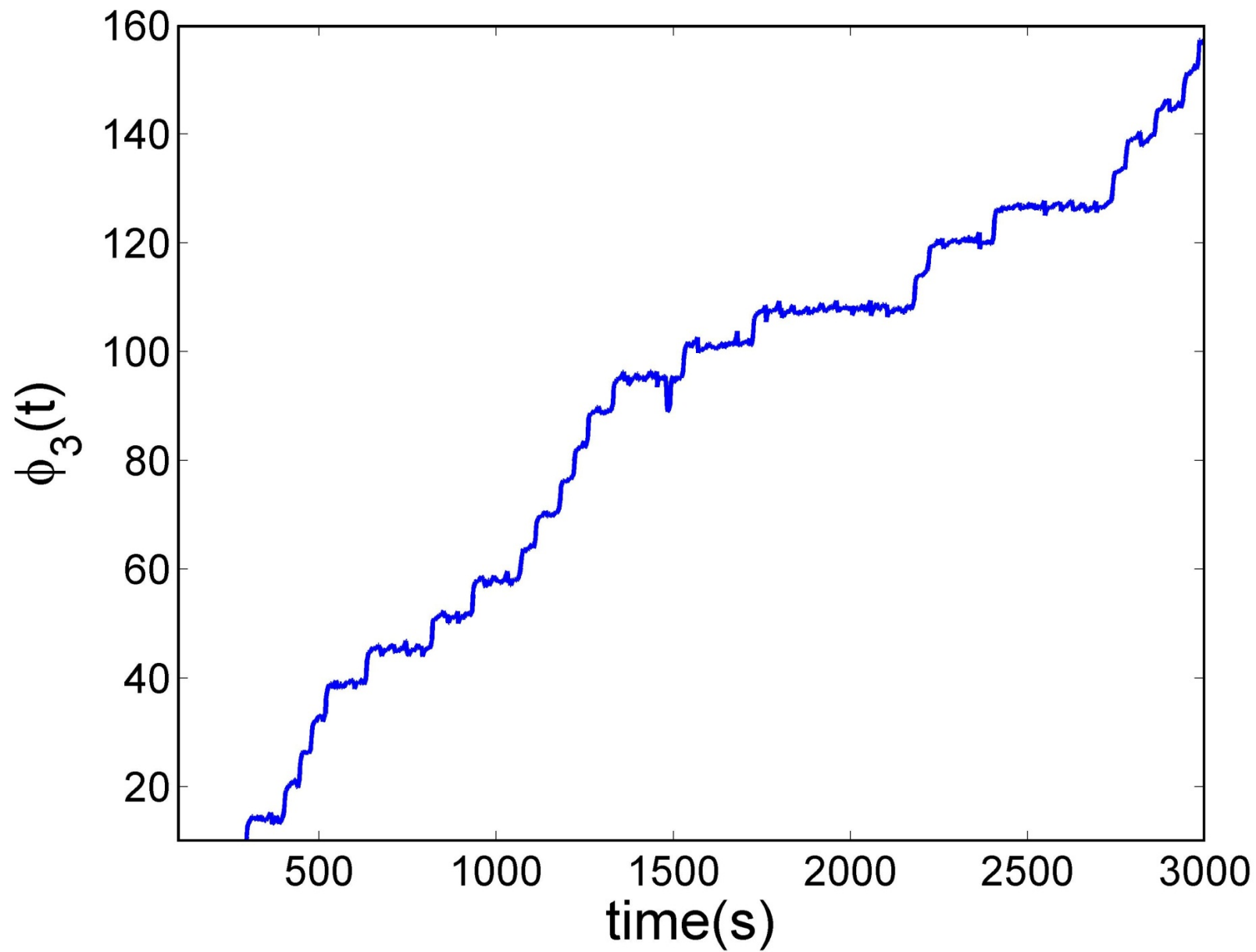


### Upper Branch

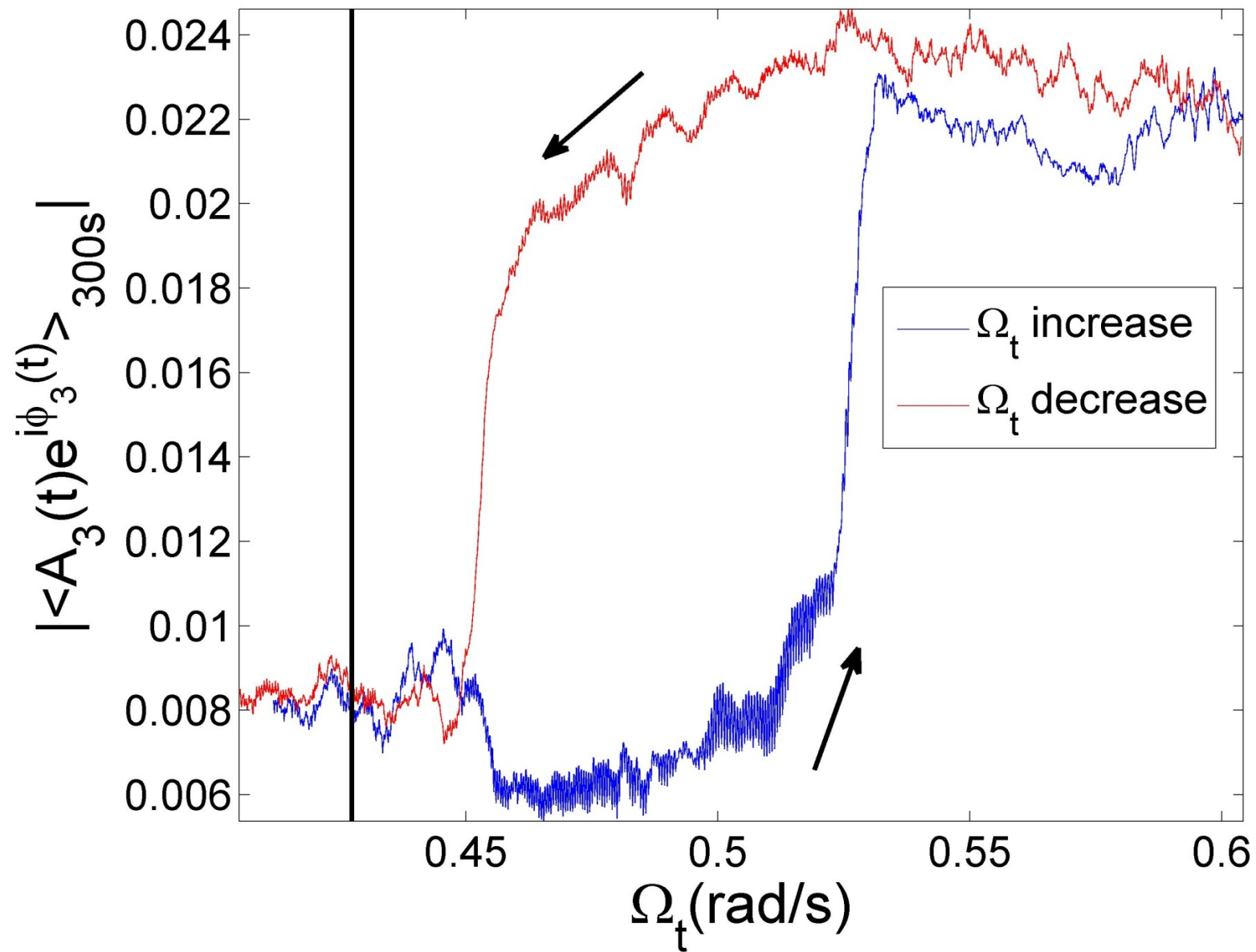


- Lower branch exhibits propagating features
- Upper branch characterized by a strong cyclonic vortex downstream of the topography
- No spontaneous transitions observed

# Sweep over $\Omega_t$

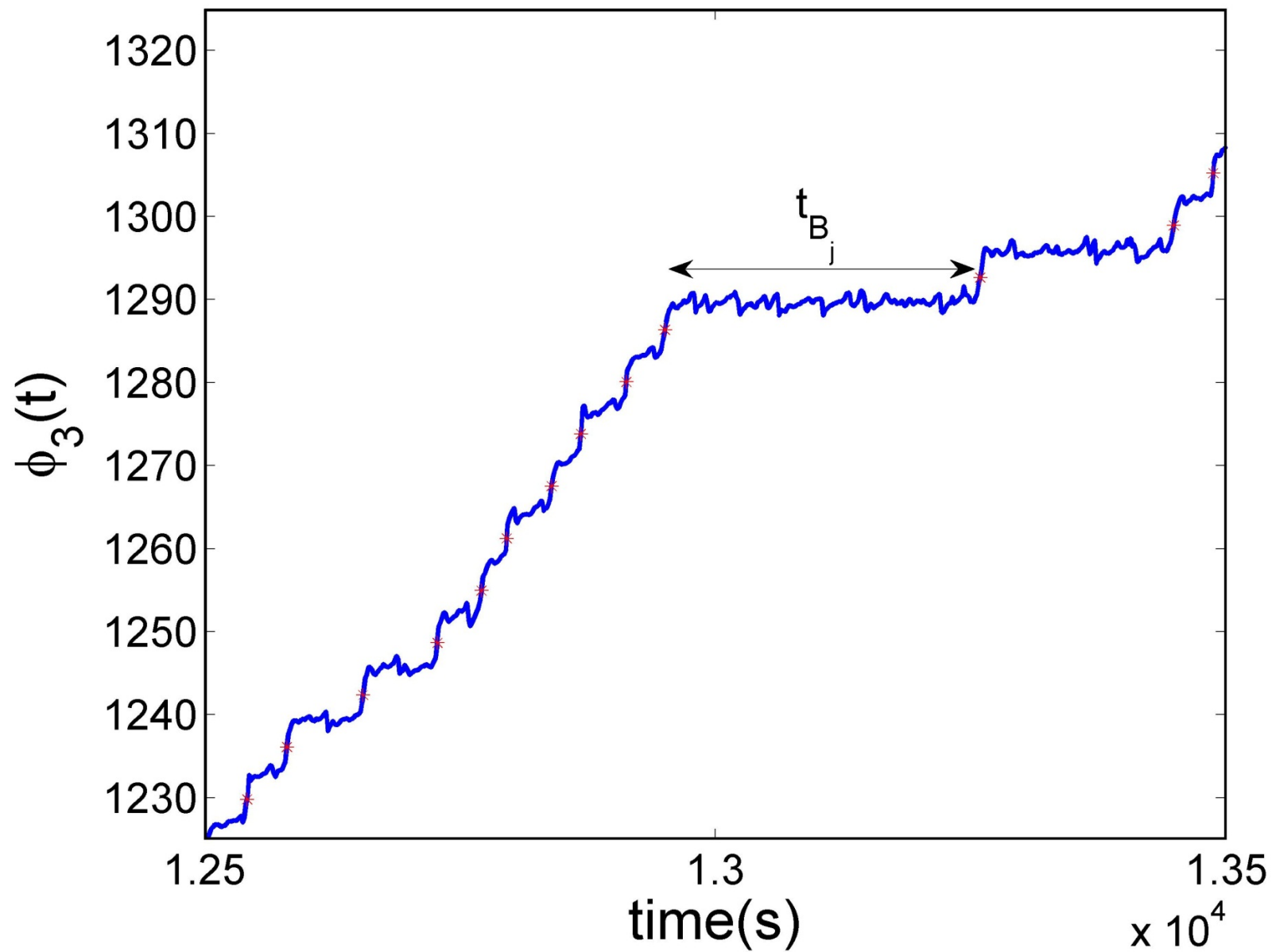


# Sweep over $\Omega_t$

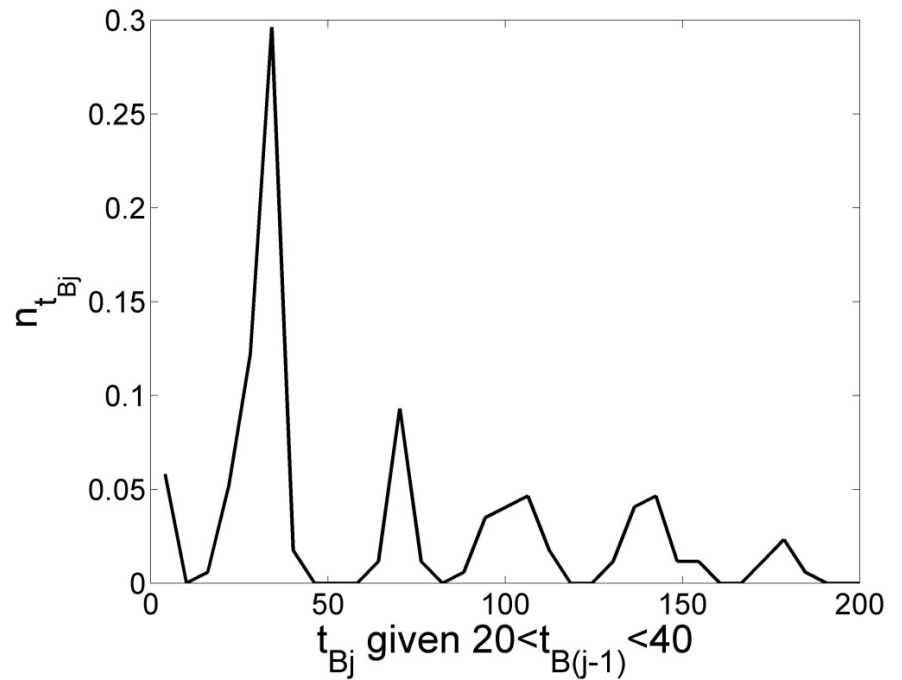
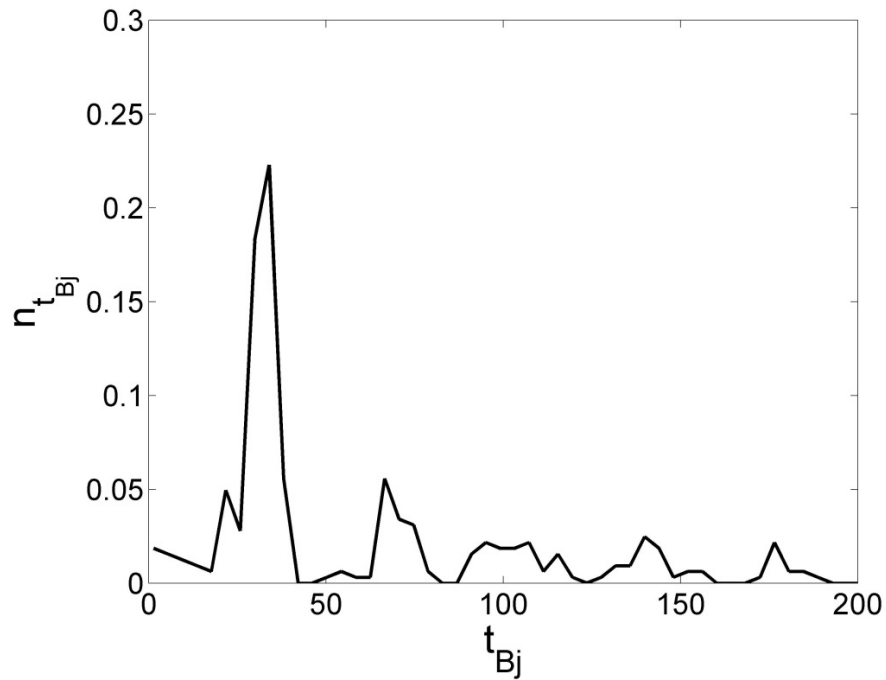




$\Omega_t = 0.43 \text{ rad/s}$



$$\underline{\Omega_t} = 0.43 \text{ rad/s}$$



- Peaks around a specific frequency and sub-harmonics
- Conditional probability indicates memorylessness

## Conclusions

- First order phase transition, and hence bistability, induced by topography in a geostrophic flow.
- Spontaneous switches not observed in the laboratory experiments. Comparisons with numerical simulations ongoing.
- “Mixed state” observed. Time spent on “blocked” state memoryless.

Thank you